College Algebra — Final Exam
MAT 140C, Spring 2024 — D. Ivanšić

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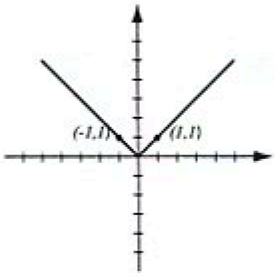
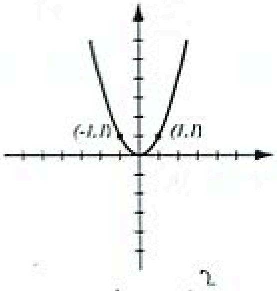
Show all your work!

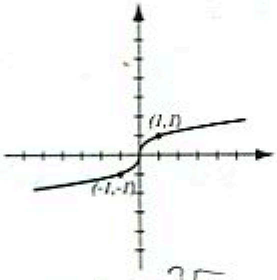
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



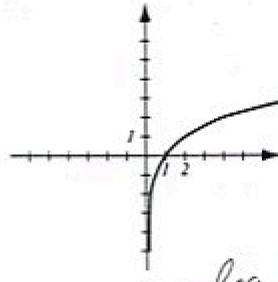
$y=|x|$



$y=x^2$



$y=\sqrt[3]{x}$



$y=\log_a x,\ a>1$

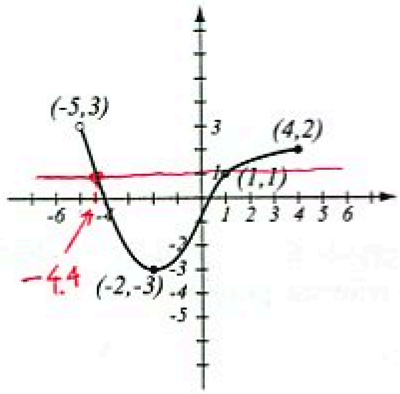
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(4)=$ 2 $\quad f(-5)=$ not defined

b) What is the domain of f? $(-5,4]$

c) What is the range of f? $[-3,3)$

d) What are the solutions of the equation $f(x)=1$? $x=-4.4,\ 1$



3. (5pts) Find the equation of the line that passes through point $(4,-2)$ and is perpendicular to the line $y=\frac{2}{3}x-1$.

Slope of given line: $\frac{2}{3}$

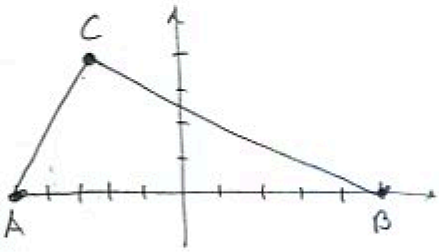
Slope of perp. line $-\frac{3}{2}$

$y-(-2)=-\frac{3}{2}(x-4)$

$y+2=-\frac{3}{2}x+6$

$y=-\frac{3}{2}x+4$

4. (7pts) Draw the triangle with vertices $A=(-5,0)$, $B=(5,0)$ and $C=(-3,4)$. Use either the Pythagorean theorem and lengths of sides or use slopes to determine if this is a right triangle.



Slopes

$AB: \frac{0-0}{5-(-5)}=0$

$BC: \frac{4-0}{-3-5}=\frac{4}{-8}=-\frac{1}{2}$

$AC: \frac{4-0}{-3-(-5)}=\frac{4}{2}=2$

opposite reciprocal so AC, AB are perp.

Pyth. theorem:

$d(A,B)=\sqrt{(5-(-5))^2+(0-0)^2}=\sqrt{10^2}=10$

$d(A,C)=\sqrt{(-3-(-5))^2+(4-0)^2}=\sqrt{4+16}=\sqrt{20}$

$d(B,C)=\sqrt{(-3-5)^2+(4-0)^2}=\sqrt{64+16}=\sqrt{80}$

$\sqrt{20}^2+\sqrt{80}^2\overset{?}{=}10^2$ yes, so triangle is right

5. (3pts) Find the domain of the function $f(x) = \dfrac{5}{x^2 - 5}$ and write it in interval notation.

Can't have $x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

$-\sqrt{5}$ $\quad$ $\sqrt{5}$

Domain: $(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, \sqrt{5}) \cup (\sqrt{5}, \infty)$

6. (6pts) Solve and write the solution in interval notation.

$|3x + 1| < 2$

$-2 < 3x+1 < 2$

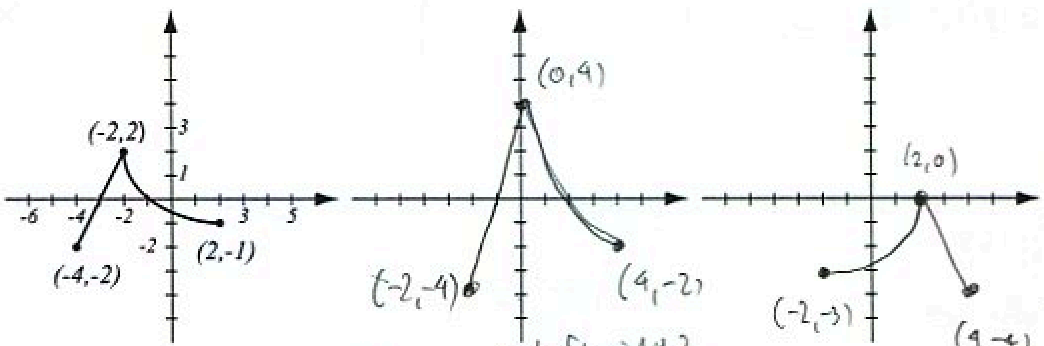
$-3 < 3x < 1$

$-1 < x < \frac{1}{3}$

-1 $\quad$ $\frac{1}{3}$

$(-1, \frac{1}{3})$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-2)$ and $f(-x) - 2$ and label all the relevant points.



$y = f(-x) - 2$

reflect in y-axis

shift down 2

$y = 2f(x-2)$

shift right 2

vert. stretch, factor 2

8. (6pts) Let $f(x) = 46e^{x+7}$. Find the formula for f^{-1}.

$y = 46e^{x+7}$

$\frac{y}{46} = e^{x+7}$ $\quad | \ln$

$\ln\frac{y}{46} = \ln e^{x+7}$

$\ln\frac{y}{46} = x + 7$

$x = \ln\frac{y}{46} - 7 = f^{-1}(y)$

9. (12pts) The quadratic function $f(x) = x^2 + 2x - 8$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) $x^2+2x-8=0$ y-int. $f(0)=-8$

$(x+4)(x-2)=0$

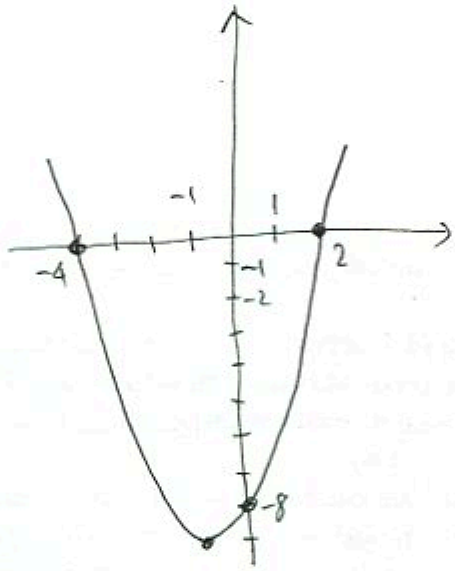
x-int: $x=-4, 2$

b) $h=-\frac{b}{2a}=-\frac{2}{2\cdot 1}=-1$

$k=f(-1)=(-1)^2+2(-1)-8$

$=1-2-8$

$=-9$



10. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 (81x^2 \sqrt[6]{y}) = \log_3 81 + \log_3 x^2 + \log_3 y^{\frac{1}{6}}$

$= 4 + 2\log_3 x + \frac{1}{6}\log_3 y$

11. (6pts) Write as a single logarithm. Simplify if possible.

$4\ln(u^{-3}v^2) - 3\ln(u^6v^2) = \ln(u^{-3}v^2)^4 - \ln(u^6v^2)^3$

$= \ln\frac{(u^{-3}v^2)^4}{(u^6v^2)^3} = \ln\frac{u^{-12}v^8}{u^{18}v^6} = \ln(u^{-12-18}v^{8-6})$

$= \ln(u^{-30}v^2) = \ln\frac{v^2}{u^{30}}$

12. (6pts) Let $f(x) = x^2 + 3x + 4$, $g(x) = \sqrt{x-2}$. Find the following (simplify where possible):

$(fg)(x) = (x^2+3x+4)\sqrt{x-2}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x-2}) = \sqrt{x-2}^2 + 3\sqrt{x-2} + 4$

$= x-2+3\sqrt{x-2}+4$

$= x+3\sqrt{x-2}+2$

13. (20pts) The polynomial $P(x) = x^2(x-3)(x+3)$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither. (Multiply out the factors if you need to.)
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) $x^2 \cdot (x)(x) = x^4$ like x^4 $\cup$

b)

zero	0	3	-3
mult.	2	1	1

y-int: $P(0) = 0$

c) $P(-x) = (-x)^2(-x-3)(-x+3)$

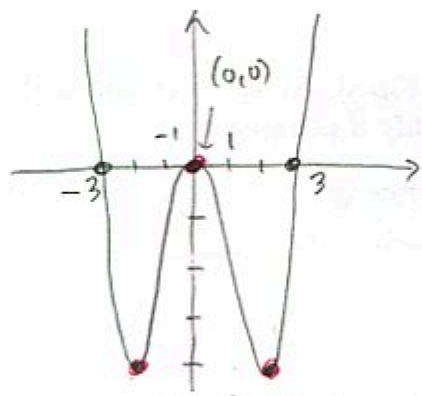
$= x^2(-1)(x+3)(-1)(x-3)$

$= x^2(x+3)(x-3)$

$= P(x)$

function is even

d)



$(-2.121319, -20.25)$ $(2.121319, -20.25)$

e) graph is symmetric about y-axis

f) three $\boxed{\text{turning points}}$ indicated on graph

14. (8pts) Solve the equation.

$x + 9 = 3 - \sqrt{60+7x}$ $\quad | -3$

$x+6 = -\sqrt{60+7x}$ $\quad |^2$

$(x+6)^2 = (-1)^2\sqrt{60+7x}^2$

$x^2+12x+36 = 60+7x$

$x^2+5x-24=0$

$(x+8)(x-3)=0$

$x=-8, \cancel{3}$

not a solution

Check:

$-8+9 \stackrel{?}{=} 3-\sqrt{60-56}$

$1 \stackrel{?}{=} 3-\sqrt{4}$ yes

$3+9 \stackrel{?}{=} 3-\sqrt{60+21}$

$12 \stackrel{?}{=} 3-\sqrt{81}$ no

15. (14pts) Kurt drives to Hopkinsville on the highway at 68mph. Due to flooding of the highway, on the way back he has to take a slower route where he averages 54mph. This route is 11 miles longer and takes him 20 minutes more to drive.
a) How long did Kurt drive to Hopkinsville?
b) How long was the slower route?

Here $\xrightarrow{d,\ 68,\ t}$ Hoptown

$\xleftarrow[d+11,\ 54,\ t+\frac{1}{3}]{}$ $(20 \text{ min} = \frac{1}{3} \text{ hr})$

$$\left.\begin{aligned} d &= 68t \\ d+11 &= 54\left(t+\tfrac{1}{3}\right) \end{aligned}\right\}$$

$68t+11 = 54\left(t+\frac{1}{3}\right)$

$68t+11 = 54t+18$

$14t = 7$

$t = \frac{7}{14} = \frac{1}{2}$ hr.

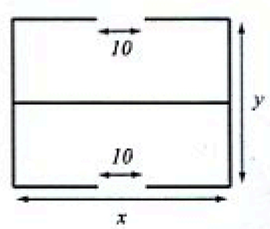
a) Kurt drove $\frac{1}{2}$ hr

b) Slower route is

$54\left(\frac{1}{2}+\frac{1}{3}\right) = \cancel{54}^{9}\,\frac{5}{\cancel{6}_{1}}$

$= 45$ miles,

16. (14pts) Jeffrey is designing a combo gas station / fast food restaurant building with 10-foot wide entrances on either side. His budget allows for 280 feet of total wall length. Jeffrey's goal is to maximize the total area of the building.
a) Express the total area of the building as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible total area, and what is the biggest possible total area?



$280 = x-10+x+x-10+2y$

$280 = 3x+2y-20$

$300 = 3x+2y$

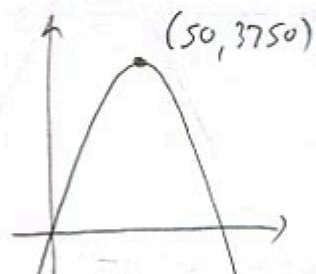
$2y = 300-3x$

$y = 150-\frac{3}{2}x$

$A = xy$

$= x\left(150-\frac{3}{2}x\right)$

$= -\frac{3}{2}x^2+150x$



Domain:

Must have

$x \geq 10$

$y \geq 0$

$150-\frac{3}{2}x \geq 0$

$\frac{3}{2}x \leq 150 \quad \mid \cdot \frac{2}{3}$

$x \leq 100$

Domain: $[10, 100]$

Dimensions:

50 by 75 ft

Max area: 3750 sq. ft

$h = -\frac{b}{2a} = -\frac{150}{2\cdot\left(-\frac{3}{2}\right)} = 50$

$k = 50\cdot\left(150-\frac{3}{2}\cdot 50\right) = 50\cdot 75 = 3750$

17. (12pts) Census data has the population of the state of Missouri as 5,595,000 in 2000 and 6,155,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2000. Graph it on paper.
b) Find the predicted population in the year 2027.

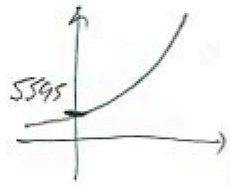
$P(t) = 5595\, e^{kt}$ in thousands

$6155 = P(20) = 5595\, e^{k \cdot 20}$

$\frac{6155}{5595} = e^{20k}$ | ln

$\ln \frac{6155}{5595} = 20k$

$k = \frac{\ln \frac{6155}{5595}}{20} = 0.00476957$



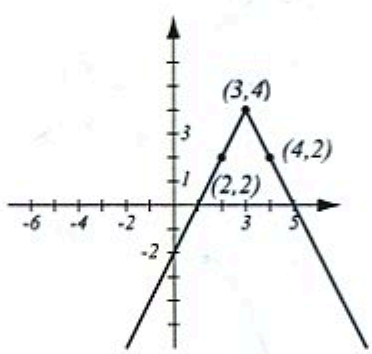
b) 2027 is $t = 27$

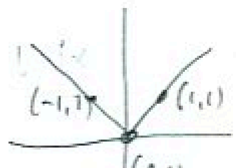
$P(27) = 5595\, e^{0.00477\ldots \cdot 27}$

$= 6363.966$

About 6,363,966 people

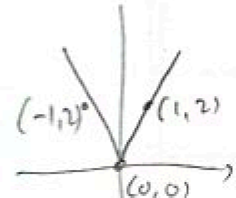
Bonus (10pts) The graph below was obtained by transformations of a graph of a standard function. Identify the standard function and the transformations and use them to write the formula for the function in the picture.





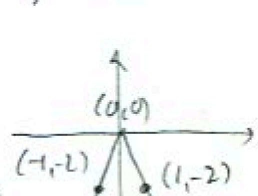
$y = |x|$

$\rightarrow$ vert stretch Factor 2



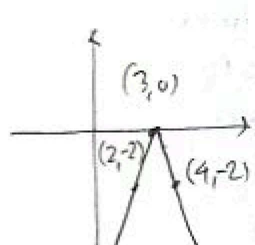
$y = 2|x|$

$\rightarrow$ reflect in x-axis



$y = -2|x|$

$\rightarrow$ right 3



$y = -2|x-3|$

up 4

$y = -2|x-3| + 4$