College Algebra — Exam 4
MAT 140C, Spring 2024 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_3 9 =$ 2 $\qquad$ $\log_4 \frac{1}{64} =$ -3 $\qquad$ $\log_b \sqrt[3]{b^5} =$ $\frac{5}{3}$ $\qquad$ $\log_{\sqrt{a}} a^3 =$ 6

$3^? = 9$ $\qquad$ $4^? = \frac{1}{64} = \frac{1}{4^3} = 4^{-3}$ $\qquad$ $b^? = \sqrt[3]{b^5} = b^{\frac{5}{3}}$ $\qquad$ $(\sqrt{a})^? = a^3$ $\quad$ $(a^{\frac{1}{2}})^? = a^3$

$\frac{1}{2} \cdot ? = 3$

$? = 6$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 0.07$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_7 0{,}07 = \frac{\ln 0{,}07}{\ln 7} = -1.366589$

3. (5pts) If $\log_a 2 = 0.3155$ and $\log_a 5 = 0.7325$, calculate the following values:

$\log_a \frac{5}{2} =$ $\log_a 5 - \log_a 2$

$= 0{,}7325 - 0{,}3155$

$= 0{,}417$

$\log_a 100 =$ $\log_a 10^2 = 2\log_a (2 \cdot 5)$

$= 2(\log_a 2 + \log_a 5)$

$= 2 \cdot (0.3155 + 0.7325) = 2 \cdot 1{,}048$

$= 2.096$

4. (4pts) Simplify.

$\log_4 4^{1-3x} =$ $1 - 3x$ $\qquad$ $e^{\ln(2a-3b)} =$ $2a - 3b$

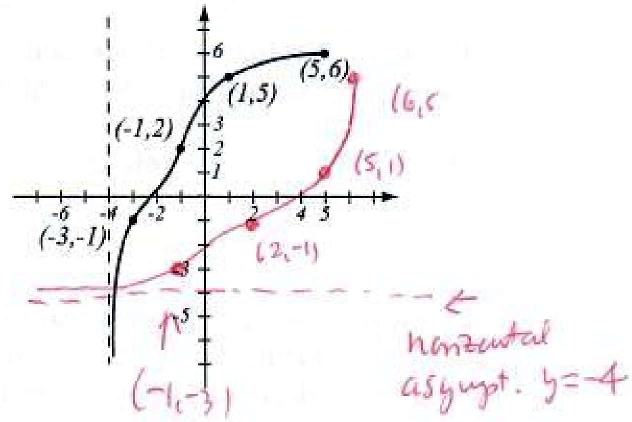
5. (8pts) If you deposit \$4,000 in an account bearing 3.4% interest, compounded daily, how much is in the account after 5 years?

$A = P(1 + \frac{r}{n})^{nt}$

$A = 4000\left(1 + \frac{0{,}034}{365}\right)^{365 \cdot 5} = 4000 \cdot 1{,}185\ldots$

$= 4741.18$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) Yes - it passes the horizontal line test

7. (9pts) Let $f(x) = \dfrac{2x-5}{x+6}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = \dfrac{2x-5}{x+6}$ solve for x

$y(x+6) = 2x-5$

$yx + 6y = 2x - 5$

$yx - 2x = -6y - 5$

$x(y-2) = -6y-5$

$x = \dfrac{-6y-5}{y-2} = \dfrac{6y+5}{2-y} = f^{-1}(y)$

Range of f = domain of f^{-1}

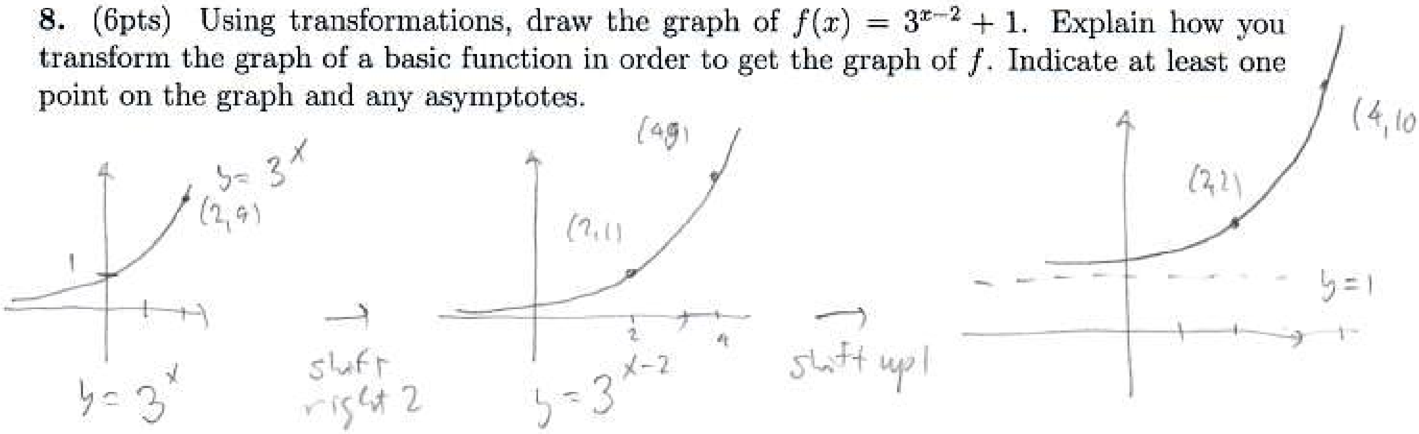
can't have $2-y=0$

$y=2$

~~all other~~ 2

$(-\infty, 2) \cup (2, \infty)$

8. (6pts) Using transformations, draw the graph of $f(x) = 3^{x-2} + 1$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_2\left(16\sqrt[4]{x^7}y^3\right) = \log_2 16 + \log_2 x^{\frac{7}{4}} + \log_2 y^3$

$= 4 + \frac{7}{4}\log_2 x + 3\log_2 y$

$\log_5 \dfrac{125x^7y^4}{y^7} = \log_5 125 + \log_5 x^7 + \log_5 y^4 - \log_5 y^7$

$= 3 + 7\log_5 x + 4\log_5 y - 7\log_5 y$

$= 3 + 7\log_5 x - 3\log_5 y$

10. (12pts) Write as a single logarithm. Simplify if possible.

$2\log_6(s^7t^4) + 3\log_6(s^{-4}t^3) = \log_6 (s^7t^4)^2 + \log_6 (s^{-4}t^3)^3$

$= \log_6 (s^{14}t^8) + \log_6 (s^{-12}t^9)$

$= \log_6 (s^{14}t^8 \cdot s^{-12}t^9) = \log_6 (s^2t^{17})$

$2\log(x^2 - x - 20) + \log(x-5) - 3\log(x+4) = \log (x^2-x-20)^2 + \log(x-5) + \log(x+4)^3$

$= \log \dfrac{(x^2-x-20)^2 \cdot (x-5)}{(x+4)^3} = \log \dfrac{((x+4)(x-5))^2(x-5)}{(x+4)^3} = \log \dfrac{(x+4)^2(x-5)^3}{(x+4)^3}$

$= \log \dfrac{(x-5)^3}{x+4}$

Solve the equations.

11. (6pts) $27^{2x+1} = 3^{1-4x}$

$(3^3)^{2x+1} = 3^{1-4x}$

$3^{6x+3} = 3^{1-4x}$

$6x + 3 = 1 - 4x$

$10x = -2$

$x = -\frac{1}{5}$

12. (8pts) $6^{x-3} = 10^{2x+1}$

apply log b/c of base 10

$\log 6^{x-3} = \log 10^{2x+1}$

$(x-3)\log 6 = 2x + 1$

$\log 6 \cdot x - 3\log 6 = 2x + 1$

$\log 6 x - 2x = 1 + 3\log 6$

$x(\log 6 - 2) = 1 + 3\log 6$

$x = \dfrac{1 + 3\log 6}{\log 6 - 2} = -2.729023$

13. (12pts) Census data has the population of the state of Missouri as 5,989,000 in 2010 and 6,155,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
b) Find the predicted population in the year 2032.

$P(t) = P_0 e^{kt}$

$P(t) = 5989 e^{kt}$

$6155 = P(10) = 5989 e^{k \cdot 10}$

$\frac{6155}{5989} = e^{10k}$ | ln

$\ln \frac{6155}{5989} = 10k$

$k = \frac{\ln \frac{6155}{5989}}{10} = 0.002739...$

b) 2032 is $t = 22$ (years since 2010)

$P(22) = 5989 \cdot e^{0.002734 \cdot 22}$

$= 6360.284616$

About 6,360,250 people

Bonus (10pts) Among the four "and" statements below, two are false regardless of which a, b, c, d are chosen, and two can be true (with the right choice of a, b, c, d). Which ones are which and why? *(Hint: use rules for working with logarithms. Note you are not asked to find a, b, c, d, so don't try. If you can find a reason why the two equations in a statement do not agree with each other, then you have found a false statement.)*

$\log_a 3 = 0.251$ and $\log_a 9 = 0.502$

$\log_a 9 = \log_a 3^2 = 2 \log_a 3$

$= 2 \cdot 0.251 = 0.502$

Ok

$\log_b 4 = 1.731$ and $\log_b 16 = 3.03$

False

$\log_b 16 = \log_b 4^2 = 2 \log_b 4$

$= 2 \cdot 1.731 = 3.462 \neq 3.03$

$\log_c 5 = 1.325$ and $\log_c(5c) = 2.021$

False

$\log_c (5c) = \log_c 5 + \underbrace{\log_c c}_{=1}$

$= 1.325 + 1 = 2.325 \neq 2.021$

$\log_d 7 = 2.513$ and $\log_d(7d) = 3.513$

$\log_d (7d) = \log_d 7 + \log_d d$

$= 2.513 + 1 = 3.513$

Ok