College Algebra — Exam 3
MAT 140C, Spring 2024 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $2i(1-3i)^2 = 2i(1^2 - 2\cdot 1\cdot 3i + (3i)^2) = 2i(1-6i+\overset{-9}{9i^2})$
$= 2i(-8-6i) = -16i - 12i^2 = 12 - 16i$

2. (5pts) $\dfrac{3+2i}{-2-i} = \dfrac{3+2i}{-2-i}\cdot\dfrac{-2+i}{-2+i} = \dfrac{-6-4i+3i+2i^2}{(-2)^2-i^2} = \dfrac{-6-i-2}{4-(-1)} = \dfrac{-8-i}{5}$

3. (4pts) Simplify and justify your answer.

$i^{111} = i^{108}\, i^3 = 1\cdot \underbrace{i\cdot i}_{-1}\cdot i = -i$

$108 = 4\cdot 27$

4. (6pts) Solve the equation by completing the square.

$x^2 - 14x = 15$

$x^2 - 2\cdot x\cdot 7 + 7^2 = 15 + 7^2$

$(x-7)^2 = 15 + 49$

$(x-7)^2 = 64$

$x - 7 = \pm 8$

$x = 7 \pm 8 = 15, -1$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|3x+2| \le 4$

$-4 \le 3x + 2 \le 4$

$-6 \le 3x \le 2 \quad | \div 3$

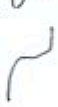
$-2 \le x \le \frac{2}{3}$

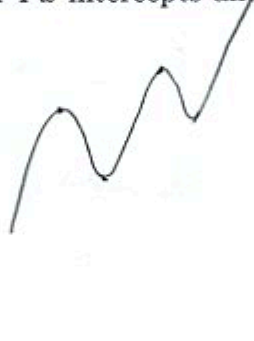
$-2 \qquad \frac{2}{3}$

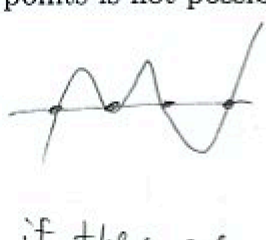
$[-2, \frac{2}{3}]$

6. (6pts) Let $P(x)$ be a polynomial of degree 5.
a) Draw a graph of P that has the maximal number of turning points.
b) Explain why a P with 4 x-intercepts and 2 turning points is not possible.

a) Max no of turning pts is 4

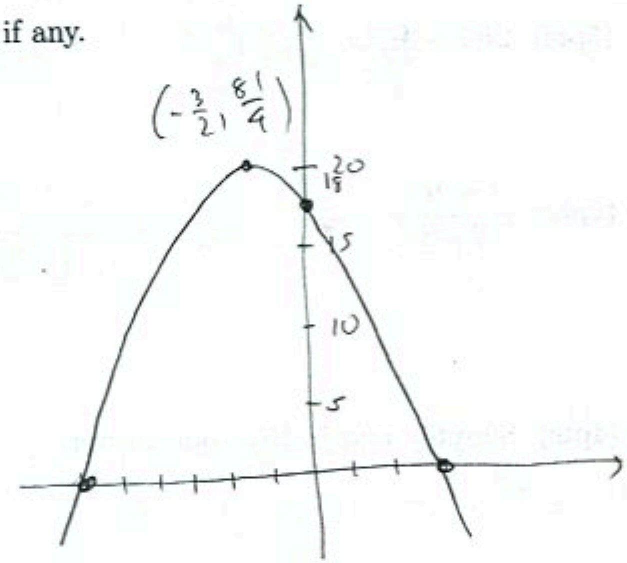
deg 5 is like 



b) 

Between every two x-int there is a turning point, so if there are 4 x-int, there are at least 3 turning points

7. (12pts) The quadratic function $f(x) = -x^2 - 3x + 18$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.



a) y-int: $f(0) = 18$

x-int: $-x^2 - 3x + 18 = 0$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6, 3$

b) vertex: $h = -\frac{-3}{2\cdot(-1)} = -\frac{3}{2}$

$k = f\left(-\frac{3}{2}\right) = -\frac{9}{4} - 3\left(-\frac{3}{2}\right) + 18 = -\frac{9}{4} + \frac{9}{2} + 18 = \frac{9}{4} + 18 = \frac{9+72}{4} = \frac{81}{4} = 20\frac{1}{4}$

Solve the equations:

8. (8pts) $\dfrac{2}{x} = \dfrac{x+1}{x-4} + \dfrac{2x-28}{x^2-4x}$

$\frac{2}{x} = \frac{x+1}{x-4} + \frac{2x-28}{x(x-4)} \quad \Big| \cdot x(x-4)$

$\frac{2}{\cancel{x}}\cancel{x}(x-4) = \frac{x+1}{\cancel{x-4}}x(\cancel{x-4}) + \frac{2x-28}{\cancel{x(x-4)}}\cancel{x(x-4)}$

$2x - 8 = x^2 + x + 2x - 28$

$x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x = -5, \cancel{4}$

$x = 4$ gives 0 in denom. of original eq.

9. (8pts) $x + \sqrt{5x - 26} = 4$

$\sqrt{5x-26} = 4 - x \quad |^2$

$5x - 26 = 4^2 - 2\cdot 4\cdot x + x^2$

$5x - 26 = 16 - 8x + x^2 \quad | -5x + 26$

$x^2 - 13x + 42 = 0$

$(x-6)(x-7) = 0$

$x = 6, 7$

Check sol: $x = 6$ $\quad 6 + \sqrt{30-26} \stackrel{?}{=} 4$

$6 + \sqrt{4} \stackrel{?}{=} 4$ no

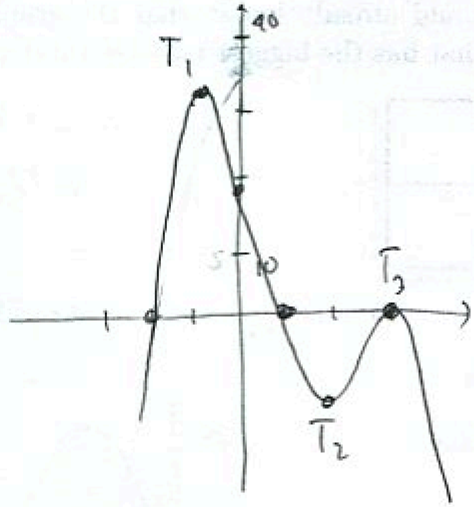
$x = 7$ $\quad 7 + \sqrt{35-26} \stackrel{?}{=} 4$

$7 + \sqrt{9} \stackrel{?}{=} 4$ no

$\boxed{\text{No solutions}}$

10. (14pts) The polynomial $f(x) = (x+2)(x-3)^2(1-x)$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

a) $x \cdot x^2(-x) = -x^4$
like $-x^4$, $\cap$



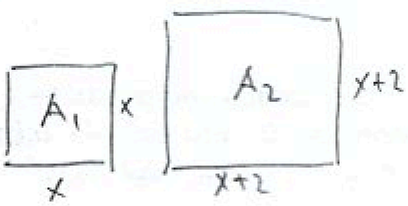
b)

zeros	-2	3	1
mult	1	2	1

y-int: $f(0) = 2\cdot(-3)^2\cdot 1 = 18$

Turning points: $T_1 = (-1, 32)$
$T_2 = (1.75, -4.394531)$
$T_3 = (3, 0)$

11. (12pts) The combined area of two separate squares is $84\,\text{cm}^2$. If the larger square has sides 2 cm longer than the smaller square, what are the lengths of the sides of the two squares?



$A_1 + A_2 = 84$
$x^2 + (x+2)^2 = 84$
$x^2 + x^2 + 2\cdot x\cdot 2 + 2^2 = 84$
$2x^2 + 4x - 80 = 0$
$x^2 + 2x - 40 = 0$ 4·1·60 4·41

$x = \dfrac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot(-40)}}{2\cdot 1} = \dfrac{-2 \pm \sqrt{164}}{2}$
$= \dfrac{-2 \pm 2\sqrt{41}}{2} = \dfrac{2(-1 \pm \sqrt{41})}{2} = -1 \pm \sqrt{41}$

x can't be negative
so $x \neq -1 - \sqrt{41}$

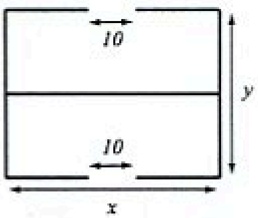
Solution: $x = -1 + \sqrt{41}$

Square side lengths
$-1 + \sqrt{41}$, $1 + \sqrt{41}$
x $\quad$ $x+2$

12. (14pts) Jeffrey is designing a combo gas station / fast food restaurant building with 10-foot wide entrances on either side. His budget allows for 350 feet of total wall length. Jeffrey's goal is to maximize the total area of the building.

a) Express the total area of the building as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible total area, and what is the biggest possible total area?



$350 = x - 10 + x + x - 10 + 2y$ $\quad 2y = 370 - 3x$

$350 = 3x - 20 + 2y \longrightarrow \quad y = 185 - \frac{3}{2}x$

$A = xy = x\left(185 - \frac{3}{2}x\right) = -\frac{3}{2}x^2 + 185x$

Domain:

Must have:

$x \geq 10$

$y > 0$

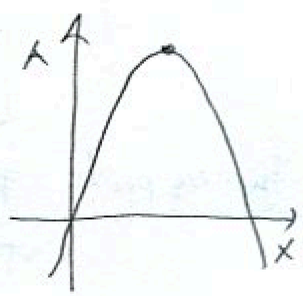
$185 - \frac{3}{2}x > 0$

$\frac{3}{2}x < 185$

$x < 185 \cdot \frac{2}{3}$

$x < 123.333333$

Domain $[10, 123.333333)$



$h = -\frac{185}{2\cdot\left(-\frac{3}{2}\right)} = \frac{185}{3} = 61.6666667$

$k = -\frac{3}{2}(61.6\ldots)^2 + 185(61.66\ldots) = 5704.166667$

x $\quad y = 185 - \frac{3}{2}61.666667$

dimensions: 61.666667 by 92.5

Max area: 5704.166667

Bonus. (10pts) Find the equation of the parabola that goes through the points $(-2, 21)$, $(0, 7)$ and $(2, 9)$. *Hint: you are looking for an equation of form $y = ax^2 + bx + c$ with unknown a, b and c. Use the three listed points to get equations for a, b and c and solve them.*

$f(x) = ax^2 + bx + c$

$21 = f(-2) = a\cdot 4 + b(-2) + c$

$7 = f(0) = a\cdot 0 + b\cdot 0 + c \Rightarrow c = 7$

$9 = f(2) = a\cdot 4 + b\cdot 2 + c$

So we rewrite other two eq.

$4a - 2b + 7 = 21$

$4a + 2b + 7 = 9$ add

$8a \quad + 14 = 30$

$8a = 16$

$a = 2$

$4\cdot 2 - 2b + 7 = 21 \quad | -7-8$

$-2b = 6$

$b = -3$

$f(x) = 2x^2 - 3x + 7$