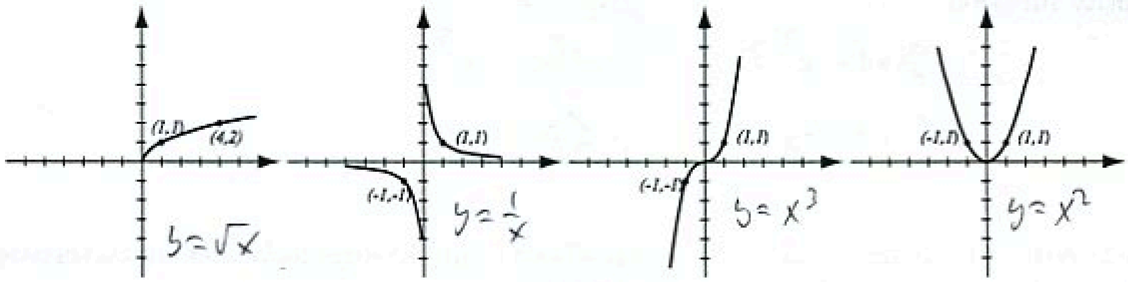
College Algebra — Exam 2	Name: Saul Ocean
MAT 140C, Spring 2024 — D. Ivanšić	*Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \dfrac{1}{\sqrt{5-x}}$, $g(x) = \sqrt{x+4}$.

Find the following (simplify where possible):

$(f-g)(0) = f(0) - g(0) = \dfrac{1}{\sqrt{5-0}} - \sqrt{0+4}$

$= \dfrac{1}{\sqrt{5}} - 2$

$(fg)(1) = f(1)\cdot g(1) = \dfrac{1}{\sqrt{5-1}} \cdot \sqrt{1+4} = \dfrac{\sqrt{5}}{\sqrt{4}} = \dfrac{\sqrt{5}}{2}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{\sqrt{5-x}}}{\sqrt{x+4}} = \dfrac{1}{\sqrt{5-x}} \cdot \dfrac{1}{\sqrt{x+4}}$

$= \dfrac{1}{\sqrt{(5-x)(x+4)}}$

$(g\circ f)(-4) = g(f(-4)) = g\left(\dfrac{1}{\sqrt{5-(-4)}}\right)$

$= g\left(\dfrac{1}{\sqrt{9}}\right) = g\left(\dfrac{1}{3}\right) = \sqrt{\dfrac{1}{3}+4} = \sqrt{\dfrac{13}{3}}$

$(f\circ g)(x) = f(g(x)) = f(\sqrt{x+4}) = \dfrac{1}{\sqrt{5-\sqrt{x+4}}}$

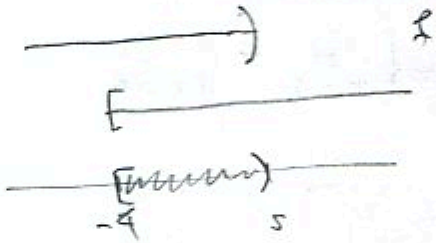
The domain of fg in interval notation

Domain f: must have $5-x>0$

$5>x$

Domain g: must have $x+4\geq 0$

$x\geq -4$



domain of fg is $[-4,5)$

3. (6pts) Consider the function $h(x) = \dfrac{1}{x^2-7}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2 - 7$ $\qquad$ $g(x) = x^2$

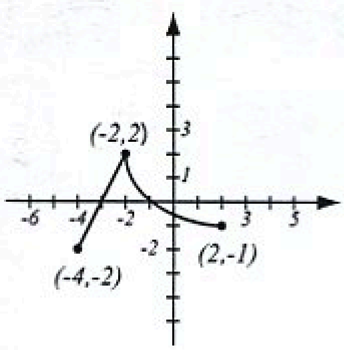
$f(x) = \frac{1}{x}$ $\qquad$ $f(x) = \frac{1}{x-7}$

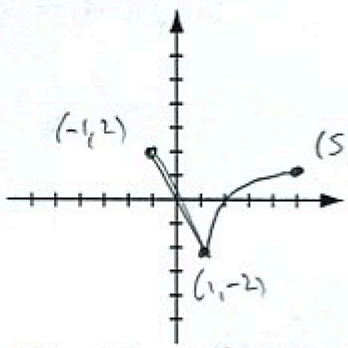
4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, stretched vertically by factor 4.
b) shape of $y = |x|$, then reflected over the x-axis and then shifted up 3.

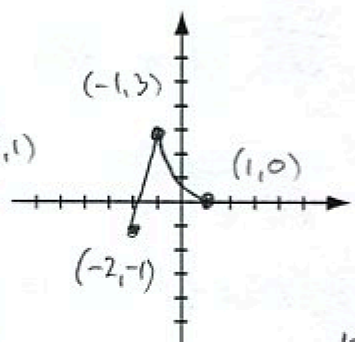
a) $\sqrt{x} \mapsto 4\sqrt{x}$

b) $|x| \mapsto -|x| \longmapsto -|x| + 3$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-3)$ and $f(2x)+1$ and label all the relevant points.







$-f(x-3)$ reflect in x-axis shift right 3

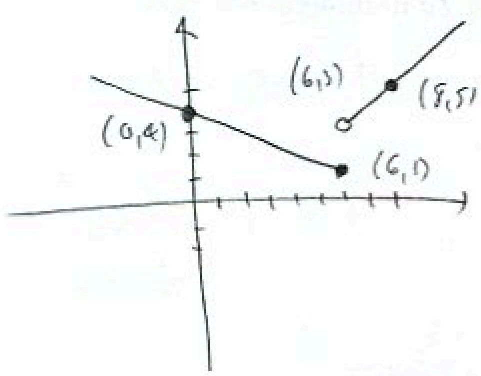
$f(2x)+1$ horiz. stretch, factor $\frac{1}{2}$ shift up 1

6. (8pts) Sketch the graph of the piecewise-defined function:

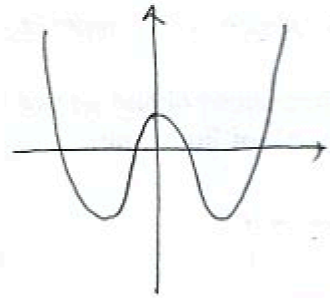
$$f(x) = \begin{cases} -\frac{1}{2}x + 4, & \text{if } x \le 6 \\ x - 3, & \text{if } x > 6 \end{cases}$$

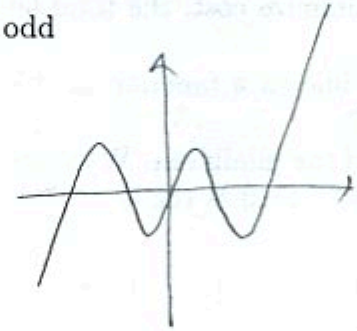
x	$-\frac{1}{2}x+4$
6	1
0	4

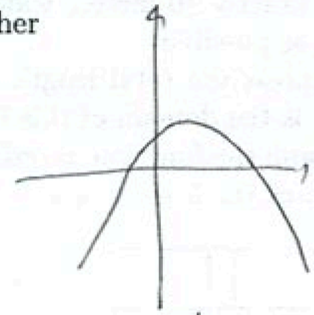
x	$x-3$
6	3
8	5



7. (8pts) In three separate coordinate systems, sketch a graph of a function that is even, odd or neither. You can draw any curve you like, as long as it has the property requested.

even 

odd 

neither 

Symm wrt y-axis   Symm. wrt origin   no symmetry

8. (20pts) Let $f(x) = \dfrac{20x}{x^2+5}$ (answer with 6 decimal points accuracy).

a) Use your graphing calculator to accurately draw the graph of f (on paper!). (If you cannot enter the function in fraction form in the calculator, make sure you put parentheses around the denominator.) Indicate units on the axes.

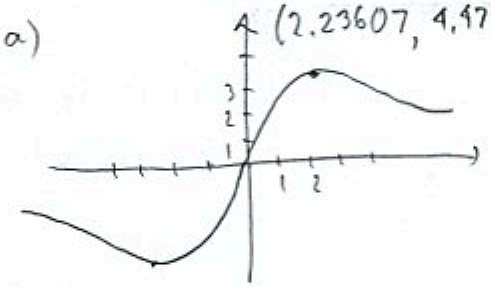
b) Determine algebraically whether the function is odd, even, or neither.

c) Verify your conclusion from b) by stating symmetry.

d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.

e) State the intervals where the function is increasing and where it is decreasing.

a) (2.23607, 4.472136)



(−2.23607, −4.472136)

b) $f(-x) = \dfrac{20(-x)}{(-x)^2+5} = \dfrac{-20x}{x^2+5} = -\dfrac{20x}{x^2+5} = -f(x)$

function is odd

c) Symmetric about the origin

d) local max is $4.472136 = f(2.23607)$

local min is $-4.472136 = f(-2.23607)$

e) increasing on $(-2.23607, 2.23607)$

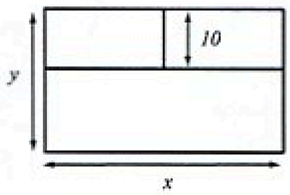
decreasing on $(-\infty, -2.23607)$

and $(2.23607, \infty)$

9. (14pts) On a beach, a rectangular section of the sea with area 4000 square meters is to be roped off with a float line and divided into three parts, as in the picture, with smaller parts exactly 10 meters wide. To minimize cost, the total length of float line has to be as small as possible.

a) Express the total length of float line as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the section that uses the least length of float line? What is the smallest total length of float line?



$A = 4000$

$xy = 4000$

$y = \frac{4000}{x}$

$L = x + x + x + y + y + 10$

$= 3x + 2y + 10$

$= 3x + 2 \cdot \frac{4000}{x} + 10$

$L(x) = 3x + \frac{8000}{x} + 10$

Domain:

Must have $x > 0$

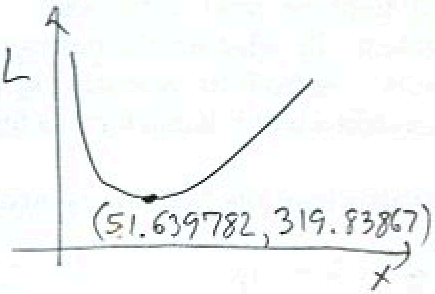
$y \geq 10$

$\frac{4000}{x} \geq 10$

$4000 \geq 10x$

$400 \geq x$

Domain is $(0, 400]$

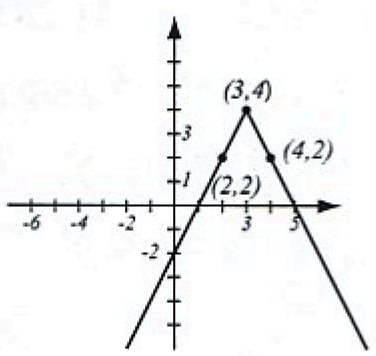


Dimensions: 51.639782 by 77.45966

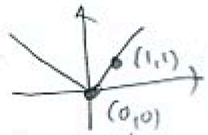
Minimal Float line length: 319.83867

Bonus. (10pts) The graph below was obtained by transformations of a graph of a standard function. Identify the standard function and the transformations and use them to write the formula for the function in the picture.

Looks like transformation of $|x|$

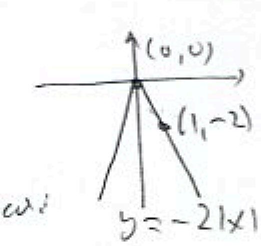


$y = -2|x-3| + 4$



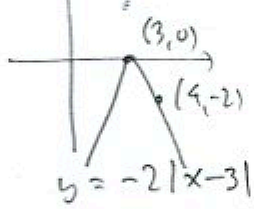
$y = |x|$ $\xrightarrow{\text{vert. stretch Factor 2}}$

$y = 2|x|$ $\xrightarrow{\text{reflect in x-axis}}$



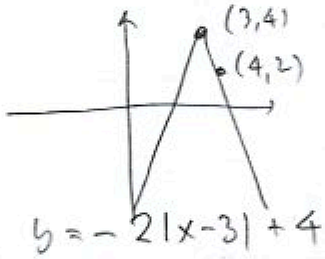
$y = -2|x|$

$\xrightarrow{\text{shift right 3}}$



$y = -2|x-3|$

$\xrightarrow{\text{shift up 4}}$



$y = -2|x-3| + 4$