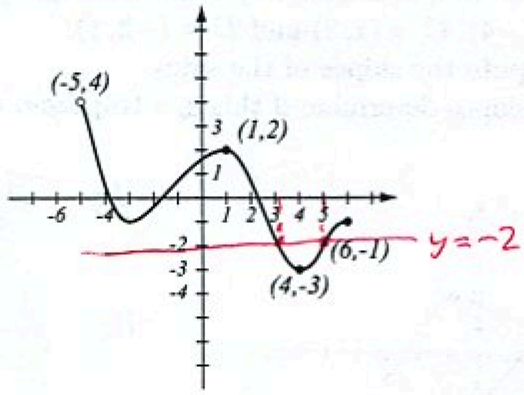
College Algebra — Exam 1	
MAT 140C, Spring 2024 — D. Ivanšić	

Name: Saul Ocean

Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(1) =$ 2 $\quad f(4) =$ −3

b) What is the domain of f? $(-5, 6]$

c) What is the range of f? $[-3, 4)$

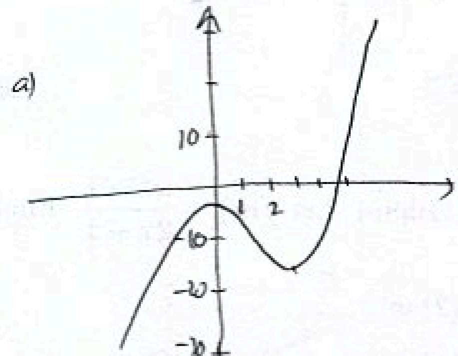
d) What are the solutions of the equation $f(x) = -2$? $x = 3, 5$

2. (12pts) Use your calculator to accurately sketch the graph of $f(x) = x^3 - 5x^2 + x - 2$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) State the range of the function in interval notation.



b) y-int: $f(0) = -2$ $\quad$ x-int: 4.879058

c) Range $= (-\infty, \infty)$

3. (5pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $y = 3x + 2$ and passes through the point $(1, -3)$. Draw the requested line.

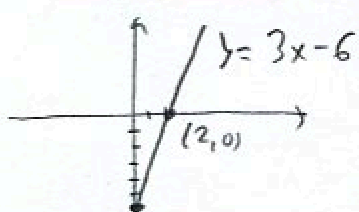
slope is 3

Our line has same slope

$y - (-3) = 3(x - 1)$

$y + 3 = 3x - 3$

$y = 3x - 6$



4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $2x - 3y = 9$ and contains the point $(1, 4)$. Draw both lines.

$2x - 3y = 9$

$-3y = -2x + 9 \quad | \div 3$

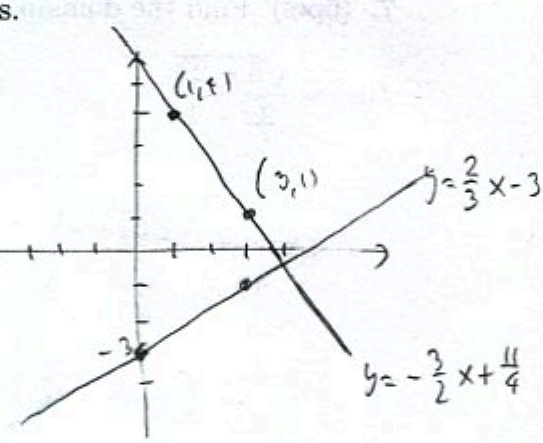
$y = \frac{-2x}{-3} + \frac{9}{-3} = \frac{2}{3}x - 3$

Slope of perp. line is $-\frac{1}{\frac{2}{3}} = -\frac{3}{2}$

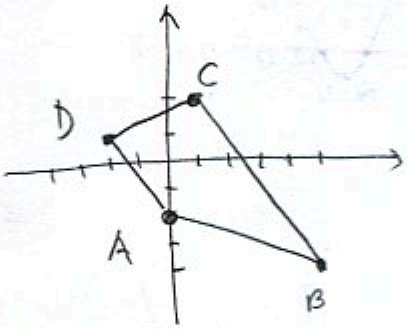
$y - 4 = -\frac{3}{2}(x - 1)$

$y = -\frac{3}{2}x + \frac{3}{2} + 4$

$y = -\frac{3}{2}x + \frac{11}{2}$



5. (8pts) In a coordinate system, draw the quadrangle with vertices $A = (0, -2)$, $B = (5, -4)$, $C = (1, 2)$ and $D = (-2, 1)$.
a) Compute the slopes of the sides.
b) Use slopes determine if this is a trapezoid (a quadrangle with two sides parallel).



$AB = \frac{-4-(-2)}{5-0} = -\frac{2}{5}$

$BC = \frac{2-(-4)}{1-5} = \frac{6}{-4} = -\frac{3}{2}$

$CD = \frac{1-2}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$

$AD = \frac{1-(-2)}{-2-0} = \frac{3}{-2} = -\frac{3}{2}$

slopes of BC and AD are same so lines are parallel – it is a trapezoid

6. (10pts) Let $f(x) = \dfrac{x^2+1}{2x-1}$. Find the following (simplify where appropriate).

$f(2) = \frac{2^2+1}{2\cdot 2-1} = \frac{5}{3}$

$f(\frac{1}{2}) = \frac{(\frac{1}{2})^2+1}{2\cdot\frac{1}{2}-1} = \frac{\frac{1}{4}+1}{1-1} = \frac{\frac{5}{4}}{0}$ not defined

$f(\sqrt{t}) = \frac{\sqrt{t}^2+1}{2\sqrt{t}-1} = \frac{t+1}{2\sqrt{t}-1}$

$f(u+3) = \frac{(u+3)^2+1}{2(u+3)-1}$

$= \frac{u^2+6u+9+1}{2u+6-1} = \frac{u^2+6u+10}{2u+5}$

7. (6pts) Find the domain of the function below and write it using interval notation.

$f(x) = \frac{\sqrt{5-2x}}{2x+4}$

Must have $5-2x \geq 0$

$-2x \geq -5$

$x \leq \frac{-5}{-2}$

$x \leq \frac{5}{2}$

Can't have $2x+4=0$

$2x=-4$

$x=-2$

-2 $\frac{5}{2}$

$(-\infty, -2) \cup (-2, \frac{5}{2}]$

8. (5pts) Solve and write the solution in interval notation.

$2 \leq 5x - 3 < 5$ $\quad | + 3$

$5 \leq 5x < 8$ $\quad | \div 5$

$1 \leq x < \frac{8}{5}$

$[1, \frac{8}{5})$

9. (10pts) The endpoints of a diameter of a circle are $(-3, 2)$ and $(1, -4)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

a) Center = midpt of $(-3,2)$ and $(1,-4)$

$= \left(\frac{-3+1}{2}, \frac{2-4}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1,-1)$

radius = distance from $(-1,-1)$ to $(1,-4)$

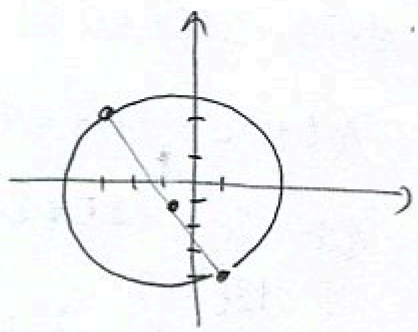
$= \sqrt{(1-(-1))^2 + (-4-(-1))^2}$

$= \sqrt{2^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

Equation of circle:

$(x-(-1))^2 + (y-(-1))^2 = \sqrt{13}^2$

$(x+1)^2 + (y+1)^2 = 13$



10. (12pts) At her coffee shop job, Esperanza can be paid in one of these ways:
A) Hourly salary of \$13.50.
B) Flat pay of \$99 for the first 10 hours, plus \$15 an hour for hours past 10.
Assuming Esperanza always works at least 10 hours per week, for which number of hours worked is pay plan A better? Solve as an inequality.

x = no. of hours worked, $x \geq 10$

$A \geq B$

$13.50x \geq 99 + 15(x-10)$

$13.5x \geq 99 + 15x - 150$

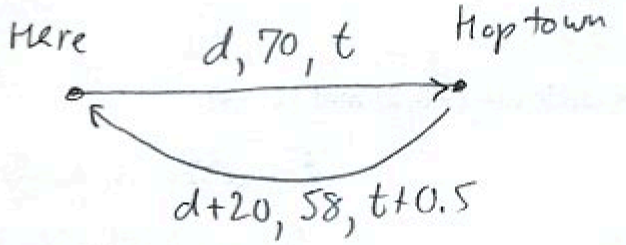
$0 \geq -51 + 1.5x$

$51 \geq 1.5x$

$x \leq \frac{51}{1.5} = 34$

If no. of hours worked is up to 18, plan A is better,

11. (14pts) Kurt drives to Hopkinsville on the highway at 70mph. Due to flooding of the highway, on the way back he has to take a slower route where he averages 58mph. This route is 20 miles longer and takes him 30 minutes more to drive.
a) How long did Kurt drive to Hopkinsville?
b) How long was the slower route?



a) Kurt drove $\frac{3}{4}$ hr to Hopkinsville

b) $58 \cdot \left(\frac{3}{4} + \frac{1}{2}\right) = \cancel{58}^{29} \cdot \frac{5}{\cancel{4}_{2}} = \frac{145}{2} = 72.5$ miles

$d = 70t$

$d + 20 = 58(t + 0.5)$

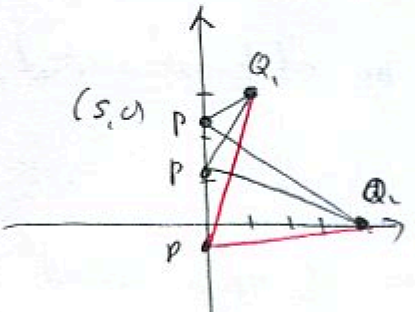
$70t + 20 = 58(t + 0.5)$

$70t + 20 = 58t + 29$

$12t = 9$

$t = \frac{9}{12} = \frac{3}{4}$ hr.

Bonus (10pts) Find a point $(0, s)$ on the y-axis that has the same distance to $(4, 0)$ and $(1, 3)$. Draw a picture. *Hint: use the distance formula to set up the equation in s that says those distances are same. Then rid the equation of square roots by squaring it, and solve it.*



Must have

distance from P to Q_1 = distance from P to Q_2

$(0, s)$ to $(1, 3)$ $\qquad$ $(0, s)$ to $(4, 0)$

$\sqrt{(1-0)^2 + (3-s)^2} = \sqrt{(4-0)^2 + (0-s)^2} \quad |^2$

$1 + (3-s)^2 = 4^2 + (-s)^2$

$1 + 9 - 6s + s^2 = 16 + s^2 \quad | -s^2$

$10 - 6s = 16$

$-6s = 6$

$s = \frac{6}{-6} = -1$

Point $(0, -1)$ has same distance to $(4, 0)$ and $(1, 3)$

(distance is $\sqrt{17}$)