College Algebra — Lecture notes MAT 140, Spring 2024 — D. Ivanšić

4.1, 4.2 Polynomial Functions and Their Graphs

Example. Draw graphs of power functions x^n for the exponents given.

x^2, x^4, x^6

x^3, x^5, x^7

Graphs of x^{even} have the same shape as x^2, graphs of x^{odd} have the same shape as x^3.

Definition. A general *polynomial* is a function of form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$$

Some terminology: $a_n x^n$ is called the *leading term*, a_n the *leading coefficient*;
if $a_n \neq 0$, n is the *degree* of the polynomial.

Example. Compare the graphs of $f(x) = 2x^3 - 3x^2 - 5x - 4$ and $g(x) = 2x^3$.

Fact. For large x or large negative x, every polynomial behaves like its leading term $a_n x^n$. This is referred to as the *end behavior* of the polynomial. Thus, graphs of polynomials look like one of the four pictures below.

Example. Draw the graph of the polynomial $f(x) = (x-2)^2(x+1)(x-4)$.

x-intercepts (also called *zeroes*) are 2, -1, 4
their corresponding *multiplicities* are 2, 1, 1 (exponents on factors related to the zeroes)

Fact. If c is a zero of a polynomial $P(x)$, then $x - c$ is a factor of $P(x)$, so $P(x) = (x-c)^k g(x)$.

Definition. If $(x-c)^{k+1}$ is not a factor of the polynomial $P(x)$, but $(x-c)^k$ is, we say the zero c has multiplicity k. Behavior of the graph at a zero depends on its multiplicity in this way:

Multiplicity of c	Graph of $P(x)$ at c
even	touches x-axis
odd	crosses x-axis

Example. Let $f(x) = (x+3)^2(x^2+7)(1-x)$. For the polynomial, find the zeroes and their multiplicities, determine end behavior and use this information to help you sketch the graph of the polynomial.

Guidelines for graphing a polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots a_2 x^2 + a_1 x + a_0$

1) Determine end behavior — for large $|x|$, it looks like $a_n x^n$.

2) Find the y-intercept, the zeroes (x-intercepts, there can be at most n zeroes) and their multiplicities.

3) Find the turning points (local minima and maxima, there can be at most $n-1$ of them).

Example. Use the guidelines to graph the polynomial $f(x) = x^4 - 4x^3 + 3x^2$.