

College Algebra — Joysheet 1
MAT 140, Fall 2024 — D. Ivanišić

Name: _____

Covers: JIT 6, 7, 11, 12, 14, 15, 17

Show all your work!

Write the sets in interval notation and sketch them on the number line.

1. (3pts) $\{x \mid -5 \leq x \leq -2\}$

2. (3pts) $\{x \mid x > -7\}$

Solve the equations.

3. (3pts) $3x + 5 = 5x - 11$

4. (4pts) $7(4 - 2a) = 3(2a - 3) + 4$

Simplify and write in standard form:

5. (4pts) $(2x - 1)^2 + x^2(3x + 1) =$

6. (4pts) $(x - 6)(x - 4) - 2x(x + 5) =$

Simplify and write the answer so all exponents are positive:

7. (2pts) $a^4(5a)^2 =$

8. (2pts) $\frac{(6v)^3}{v^5} =$

9. (3pts) $x^5(x^{-1}y^3)^3 y^{-10} =$

10. (5pts) $(u^5v^{-3})^2(3u^4v^{-2})^3 =$

11. (7pts) $\frac{(12x^3y^5)^2}{(2x^4y^{-1})^4} =$

Factor the following.

12. (4pts) $x^2 - 2x - 15 =$

13. (4pts) $x^2 - 14x + 24 =$

Use the *ac*-method or another method to factor. Show how you got your answer.

14. (6pts) $3x^2 - 10x - 8 =$

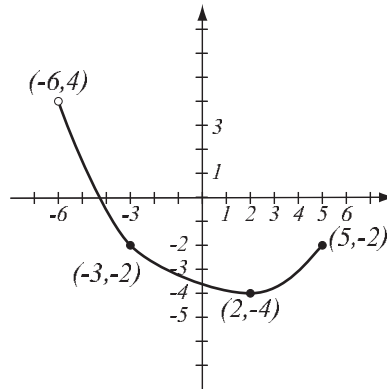
15. (6pts) $4x^2 + 4x - 15 =$

1. (11pts) Draw the quadrilateral with vertices $A = (0,0)$, $B = (4,2)$, $C = (3,4)$ and $D = (-1,2)$ in the coordinate plane.
- a) Compute the lengths of all sides of the quadrilateral and find its perimeter.
 - b) Find the length of the diagonal AC .
 - c) Is this quadrilateral a rectangle? (To see this, you essentially have to check that triangle ABC is a right triangle.)

2. (8pts) Find the equation of the circle, if its center is $(-2, -4)$ and the point $(0, -1)$ is on the circle. Draw the circle.

3. (8pts) Use the graph of the function f at right to answer the following questions.

- a) Find $f(-3)$ and $f(6)$.
- b) What is the domain of f ?
- c) What is the range of f ?
- d) What are the solutions of the equation $f(x) = -3$?



4. (12pts) The function $f(x) = 3\sqrt{x^2 - 6x + 5} - 15$ is given.

a) Use your calculator to accurately draw its graph. Draw the graph here, and indicate units on the axes.

b) Find all the x - and y -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (11pts) Find the domain of each function and write it using interval notation.

$$f(x) = \sqrt{x} + \frac{4}{3x - 8}$$

$$g(x) = \frac{x + 2}{x^2 - 3x + 2}$$

6. (10pts) Let $h(x) = \frac{x^2 + x - 9}{2x - 8}$. Find the following (simplify where appropriate).

$$h(-1) =$$

$$h(4) =$$

$$h(b^3) =$$

$$h(u + 3) =$$

5. (12pts) The water bill for a household was \$45.35 in a month when it used 25 hundred-gallons of water. In another month, it used 46 hundred-gallons and was billed \$57.11.

a) Assuming that water cost $C(x)$ is a linear function of the amount of water x used (in hundred-gallons), write a formula for $C(x)$.

b) What is the cost if no water is used during a month? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A market researcher is considering the average price-per-pound of whole chicken over the years. Below is the data for July prices of the corresponding year. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between x and P . Draw the line on the graph.

c) Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.

d) Find coefficient of correlation r . How strong is the linear relationship between x and P ?

e) What average price-per-pound of whole chicken can we expect in July of 2027?

x	P
2014	1.50
2016	1.45
2018	1.52
2020	1.71
2022	1.88
2024	1.98

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $-3 \leq 4x + 1 < 6$

2. (7pts) $5 - 3x < 1$ or $8 - x > 9$

3. (6pts) Find the domain of the function in interval notation: $f(x) = \frac{\sqrt{4x - 6}}{3x - 10}$.

4. (14pts) For her daughter's quinceañera, Valeria is planning to rent an event venue. She is considering Party Deck, which charges \$150 per event plus \$30 per hour, or Fiesta Bay, which charges \$225 per event, which includes two hours, and then \$20 per hour for every hour after the first two. Valeria's party will last at least two hours. For which number of hours is Fiesta Bay the better option for her? Solve as an inequality.

5. (14pts) A car enters a highway, driving 67 mph. At the same time, 27 miles down the road, a truck enters the same highway, and travels at 54 mph in the same direction as the car.

a) How long did it take the car to catch up with the truck?

b) How far did the car travel along the highway before it caught up with the truck?

6. (14pts) Politicians Chapelle, Rock and Silverman are counting their campaigns' funds. Silverman has twice as much as Rock, plus half a million. Chapelle has exactly as much as the difference of Silverman's and Rocks funds. Together, they have 9 million dollars. How much does each politician have in campaign funds?

1. (10pts) Use your calculator to accurately sketch the graph of the function

$f(x) = \frac{x^3 - 3x^2 + 9}{x^2 - 2x + 2}$. (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \frac{x}{3x - 1}$, $g(x) = \sqrt{2x + 7}$. Find the following (simplify where possible):

$$(f + g)(9) = \qquad \qquad \qquad \frac{f}{g}(-3) =$$

$$\frac{g}{f}(x) = \qquad \qquad \qquad (g \circ f)(1) =$$

$$(f \circ g)(x) =$$

The domain of $(f + g)(x)$ in interval notation

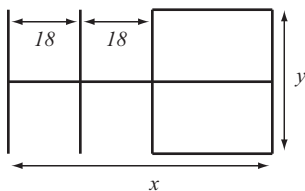
3. (8pts) Consider the function $h(x) = \frac{3}{\sqrt{x+5}}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} \frac{1}{2}x - 2, & \text{if } x \leq -2 \\ -2x + 1 & \text{if } -2 < x < 3. \end{cases}$$

5. (14pts) A paving company is planning a 3000-square-foot building consisting of office space and four bays for equipment, each 18 feet wide. The company wishes to minimize the construction cost, which is same as minimizing the total length of the walls.

- a) Express the total length of the walls as a function of the length of one of the sides x . What is the domain of this function?
- b) Graph the function in order to find the minimum. What are the dimensions of the building for which the total length of the walls is minimal? What is the minimal wall length?



1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - 7x$$

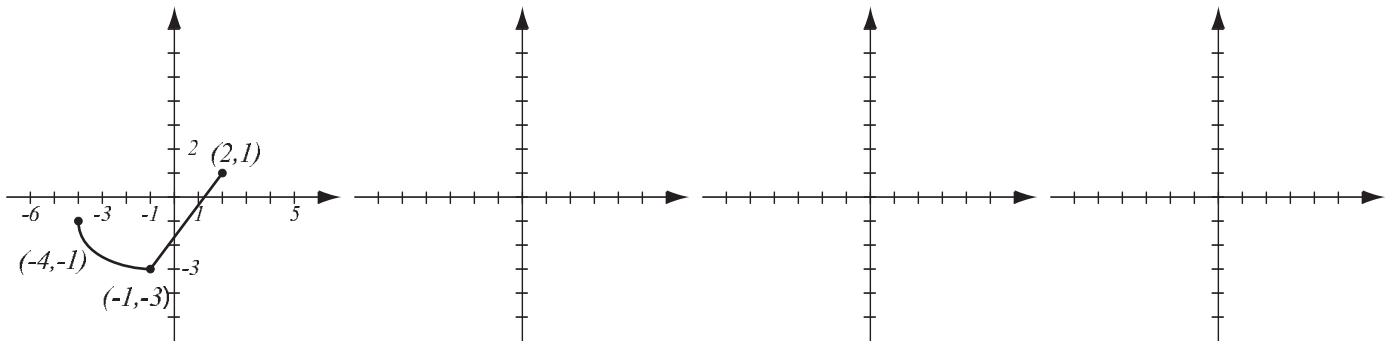
$$g(x) = x^3 - 4x^2 + 2$$

$$h(x) = 6|x| - x^2 + 3$$

2. (16pts) Draw the graphs of $f(x) = 2\sqrt{x-3}$ and $g(x) = -\left(\frac{x}{3}\right)^3 + 2$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of $y = x^2$, shifted down 3 units,
 - shape of $y = \frac{1}{x}$, reflected about the y -axis, then shifted right 7
 - shape of $y = |x|$, reflected about the x -axis, then stretched vertically by factor 2, then shifted left 4 units.

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x - 2)$, $2f(-x)$ and $f(2x) + 3$ and label all the relevant points.



Use formulas to expand:

1. (4pts) $(4u - 3v^2)(4u + 3v^2) =$

2. (4pts) $(x - 5y)^2 =$

3. (5pts) $(a^2 + 2b^2)^2 =$

4. (8pts) Compute expressions with fractions by hand.

$$\frac{6}{35} \cdot \frac{21}{8} =$$

$$\frac{3}{14} \div \frac{27}{8} =$$

$$\frac{1}{4} + \frac{7}{12} =$$

$$\frac{5}{18} - \frac{11}{24} =$$

Multiply or divide the rational expressions.

5. (7pts) $\frac{x^2 - 9x + 14}{10x - 70} \cdot \frac{2x + 6}{x^2 + 3x - 10} =$

6. (7pts) $\frac{2x^2 - 32}{2x^2 - 3x - 20} \div \frac{x + 4}{6x + 15} =$

Add or subtract the rational expressions.

7. (6pts) $\frac{2x^2 + 11x - 6}{x^2 - x - 20} - \frac{2x + 1}{x - 5} =$

8. (8pts) $\frac{14}{3x^2 - 4x - 15} - \frac{9}{x^2 + 3x - 18} =$

Simplify the following expressions, assuming all variables are positive.

9. (3pts) $\sqrt{50x^7y^3} =$

10. (4pts) $\sqrt[3]{18u^4v^7} \sqrt[3]{3u^2v^3} =$

11. (4pts) $(2 + 3\sqrt{3})(5 - \sqrt{12}) =$

College Algebra — Joysheet 8
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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(2 - 5i)^2 - 3 + 4i =$

2. (6pts) $\frac{1 + i}{5 + 2i} =$

3. (4pts) Simplify and justify your answer.

$i^{409} =$

4. (8pts) The number of crates of oranges (in thousands) at a warehouse is described by the function $N(x) = x^2 - 18x + 83$, where x is the number of days after April 27th.

a) On what dates did the warehouse have 51 thousand crates?

b) On what date did the number of crates of oranges reach its minimum?

5. (8pts) Solve the equation: $x^4 + 3x^2 - 28 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 12x + 40 = 0$

7. (12pts) The quadratic function $f(x) = x^2 + 3x - 18$ is given. Do the following without using the calculator.

- a) Find the x -intercepts of its graph, if any. Find the y -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

8. (12pts) An insurance company has contracted with a physical therapy office for services. On a base load of 30 patients per day, the office is paid \$100 per patient. For every additional patient above 30, the office is paid \$1 less per patient because the resources are better utilized, and the total number of daily patients cannot exceed 60, because it would negatively affect care quality. On one day, the physical therapy office was paid \$3649 for services. How many patients did it see on that day?

College Algebra — Joysheet 9
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Covers: 3.3, 3.4, 3.5 Show all your work!

1. (4pts) Solve the equation.

$$|5x - 1| = 11$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x - 3| < 7$$

$$|2x - 3| \geq 4$$

Solve the equations:

3. (8pts) $\frac{2x - 8}{x - 7} + \frac{x - 47}{x^2 - 4x - 21} = \frac{x + 8}{x + 3}$

4. (8pts) $x - 1 = 5 + \sqrt{24 - 4x}$

5. (14pts) An apple is thrown upwards from a balcony at height 20 meters with initial velocity 10 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 10t + 20$.

a) Sketch the graph of the height function.

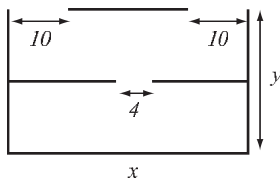
b) When does the apple reach its greatest height, and what is that height?

c) When does the apple fall to the ground?

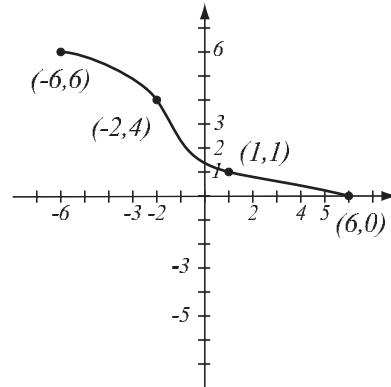
6. (14pts) Shane is building a cabin with two rooms that has two big sliding doors 10 feet wide and a door 4 feet wide. He has budgeted for 150 ft of walls and wishes to maximize the area of the cabin.

a) Express the total area of the cabin as a function of one of the sides of the rectangle. What is the domain of this function?

b) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the cabin that has the greatest total area? What is the greatest total area possible?



1. (6pts) The graph of a function f is given.
- Is this function one-to-one? Justify.
 - If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points.



2. (12pts) Let $f(x) = \frac{5x}{x-3}$. Find the formula for f^{-1} . Find the ranges of f and f^{-1} .

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_4 64 =$$

$$\log_3 \frac{1}{81} =$$

$$\log_7 \sqrt[5]{49} =$$

$$\log_{\sqrt[3]{a}} a^2 =$$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_{13} 0.26$ with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$5,000 and annual interest rate of 4.71%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of $f(x) = \ln(7 - 2x)$.

7. (8pts) If an object is launched upwards with initial velocity V meters per second, the greatest altitude it achieves in meters is given by $A = \frac{V^2}{20}$.

- Find greatest altitude achieved if launch velocity is 18 and 30 meters per second .
- Find a formula for the inverse function and explain what it represents.
- What initial velocity is needed to have the object reach 180 and 300 meters?

8. (7pts) Using transformations, draw the graph of $f(x) = e^{-x} + 1$. Explain how you transform the graph of a basic function in order to get the graph of f . Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If $\log_a 5 = 1.1610$ and $\log_a 6 = 1.2925$, calculate:

$$\log_a \frac{6}{5} =$$

$$\log_a 150 =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_8 (512x^7y^2) =$$

$$\log \frac{\sqrt{x^5y^6}}{100x^2z^4} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$4 \log_7(2x^3) + 2 \log_7 y^3 - 3 \log_7(4x^4) =$$

$$3 \log_2(x - 5) + 2 \log_2(x + 3) - 2 \log_2(x^2 - 2x - 15) =$$

4. (3pts) Simplify. $\ln e^{\sqrt{x}+1} =$ $6^{\log_6(x^2+3x)} =$

Solve the equations.

5. (5pts) $3^{2x-2} = 27^{4x-5}$

6. (7pts) $7^{x+1} = e^{3x-2}$

7. (5pts) A manufacturer buys a machine for \$43,000. The machine's value decreases by 9% every year, so after t years the value of the machine is given by the function $V(t) = 43 \cdot (0.91)^t$ (in thousands). When will the machine be worth \$15,000?

8. (12pts) According to census data, Atlanta, GA, had 417,000 inhabitants in 2000 and 499,000 in 2020. Assume the population of Atlanta grows exponentially.

- Write the function describing the number $P(t)$ of people in Atlanta t years after 2000. Then find the exponential growth rate for this population.
- Graph the function.
- According to this model, when will the population reach 600,000?