

1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - 7x$$

$$\begin{aligned} f(-x) &= (-x)^3 - 7(-x) \\ &= -x^3 + 7x \\ &= -(x^3 - 7x) = -f(x) \end{aligned}$$

odd function

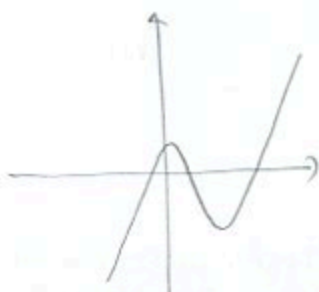


symmetric wrt origin

$$g(x) = x^3 - 4x^2 + 2$$

$$\begin{aligned} g(-x) &= (-x)^3 - 4(-x)^2 + 2 \\ &= -x^3 - 4x^2 + 2 \neq g(x) \\ &\neq -g(x) \end{aligned}$$

neither

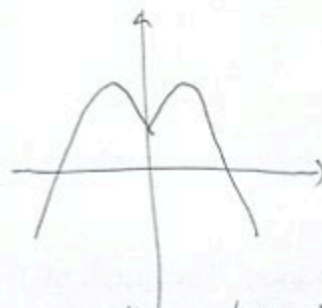


no symmetry

$$h(x) = 6|x| - x^2 + 3$$

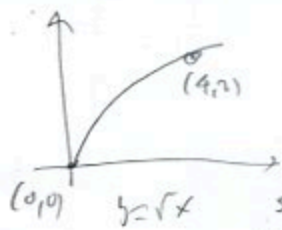
$$\begin{aligned} h(-x) &= 6|-x| - (-x)^2 + 3 \\ &= 6|x| - x^2 + 3 = h(x) \end{aligned}$$

even function



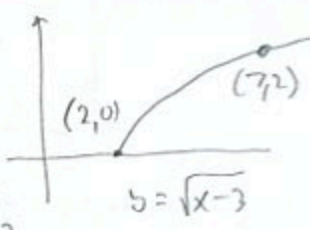
symmetric wrt y-axis

2. (16pts) Draw the graphs of  $f(x) = 2\sqrt{x-3}$  and  $g(x) = -\left(\frac{x}{3}\right)^3 + 2$  using transformations. Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.



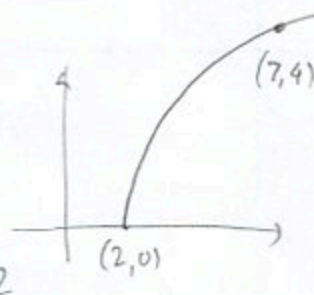
$$y = \sqrt{x}$$

shift right 3

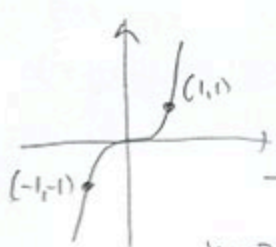


$$y = \sqrt{x-3}$$

vertical stretch, factor=2



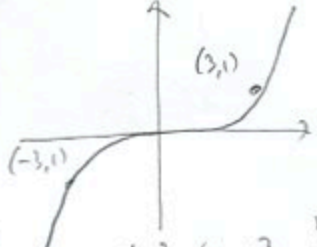
$$y = 2\sqrt{x-3}$$



$$y = x^3$$

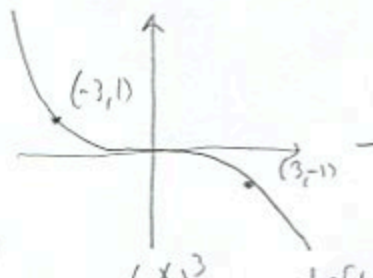
horizontal stretch

$$\text{Factor} = \frac{1}{3} = 3$$



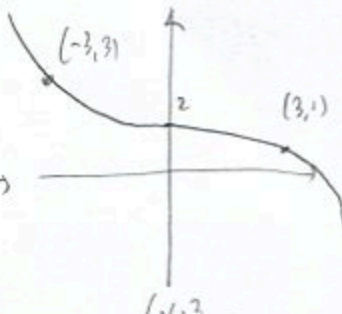
$$y = \left(\frac{x}{3}\right)^3 = \left(\frac{1}{3}x\right)^3$$

reflected in x-axis



$$y = -\left(\frac{x}{3}\right)^3$$

shift up 2



$$y = -\left(\frac{x}{3}\right)^3 + 2$$

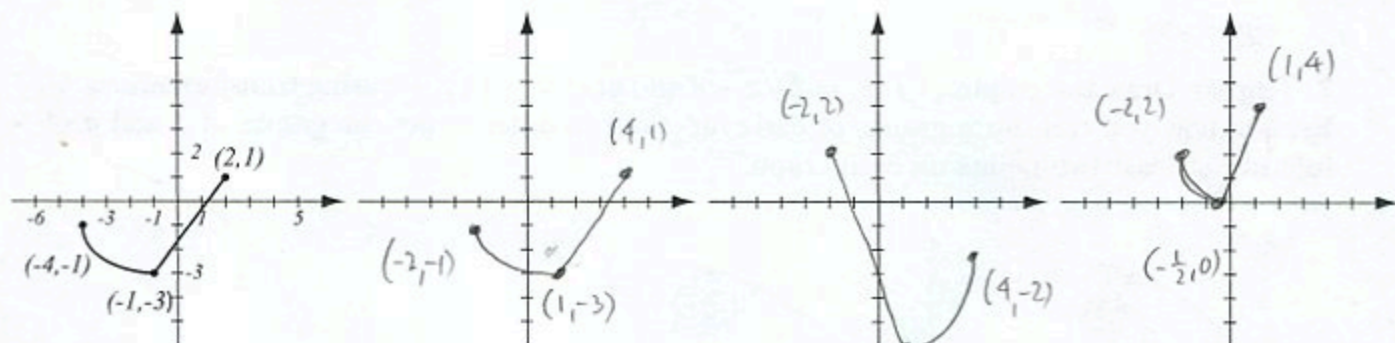
3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of  $y = x^2$ , shifted down 3 units,
  - shape of  $y = \frac{1}{x}$ , reflected about the  $y$ -axis, then shifted right 7
  - shape of  $y = |x|$ , reflected about the  $x$ -axis, then stretched vertically by factor 2, then shifted left 4 units.

a)  $x^2 \rightsquigarrow x^2 - 3$        $y = x^2 - 3$

b)  $\frac{1}{x} \rightsquigarrow \frac{1}{-x} = -\frac{1}{x} \rightsquigarrow -\frac{1}{x-7}$        $y = -\frac{1}{x-7}$

c)  $|x| \rightsquigarrow -|x| \rightsquigarrow 2(-|x|) = -2|x| \rightsquigarrow -2|x+4|$        $y = -2|x+4|$

4. (13pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(x-2)$ ,  $2f(-x)$  and  $f(2x)+3$  and label all the relevant points.



shift right 2

$$x \mapsto x+2$$

$$y \mapsto y$$

reflect in  $y$ -axis

vertical stretch, factor 2

$$x \mapsto -x$$

$$y \mapsto 2y$$

horiz stretch, factor  $\frac{1}{2}$

shift up 3

$$x \mapsto x \cdot \frac{1}{2}$$

$$y \mapsto y+3$$