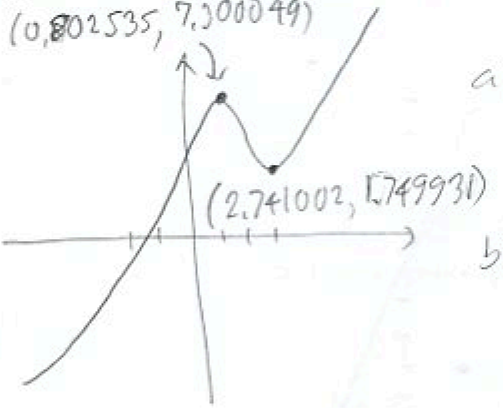
College Algebra — Joysheet 5 **MAT 140, Fall 2024 — D. Ivanšić**	**Name:** Saul Ocean *Covers: 2.1, 2.2, 2.3 Show all your work!*

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{x^3 - 3x^2 + 9}{x^2 - 2x + 2}$. (When entering function into calculator, don't forget to put parentheses around numerator and denominator if the calculator doesn't have fractional notation.) Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.



a) Local max is $7.300049 = f(0.802535)$

Local min is $1.749931 = f(2.741002)$

b) Increasing on $(-\infty, 0.802535)$ and $(2.741002, \infty)$

Decreasing on $(0.802535, 2.741002)$

2. (20pts) Let $f(x) = \dfrac{x}{3x-1}$, $g(x) = \sqrt{2x+7}$. Find the following (simplify where possible):

$(f+g)(9) = f(9) + g(9) = \dfrac{9}{27-1} + \underbrace{\sqrt{2\cdot 9+7}}_{25} = \dfrac{9}{26} + 5 = \dfrac{139}{26}$

$\dfrac{f}{g}(-3) = \dfrac{f(-3)}{g(-3)} = \dfrac{\frac{-3}{-9-1}}{\sqrt{2(-3)+7}} = \dfrac{\frac{3}{10}}{\sqrt{1}} = \dfrac{3}{10}$

$\dfrac{g}{f}(x) = \dfrac{\sqrt{2x+7}}{\frac{x}{3x-1}} = \dfrac{\sqrt{2x+7}}{1} \cdot \dfrac{3x-1}{x} = \dfrac{(3x-1)\sqrt{2x+7}}{x}$

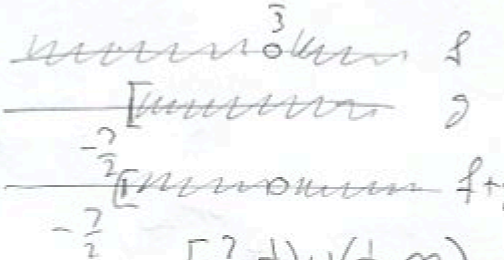
$(g \circ f)(1) = g(f(1)) = g\left(\dfrac{1}{3-1}\right) = g\left(\dfrac{1}{2}\right) = \sqrt{2\cdot\frac{1}{2}+7} = \sqrt{\underset{4\cdot 2}{8}} = 2\sqrt{2}$

$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{2x+7}\right) = \dfrac{\sqrt{2x+7}}{3\sqrt{2x+7} - 1}$

The domain of $(f+g)(x)$ in interval notation

Domain f: Can't have $3x-1=0$, $3x=1$, $x=\frac{1}{3}$

Domain g: Must have $2x+7 \geq 0$, $2x \geq -7$, $x \geq -\frac{7}{2}$



$[-\frac{7}{2}, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

3. (8pts) Consider the function $h(x) = \dfrac{3}{\sqrt{x+5}}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = \sqrt{x+5}$ $\qquad\qquad$ $g(x) = x+5$

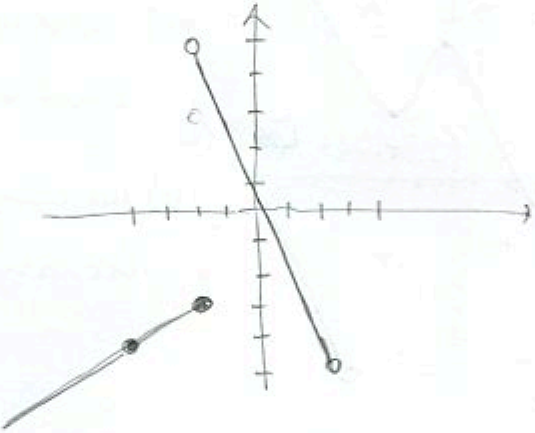
$f(x) = \dfrac{3}{x}$ $\qquad\qquad$ $f(x) = \dfrac{3}{\sqrt{x}}$

4. (8pts) Sketch the graph of the piecewise-defined function:

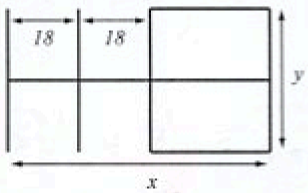
$$f(x) = \begin{cases} \frac{1}{2}x - 2, & \text{if } x \leq -2 \\ -2x + 1 & \text{if } -2 \leq x < 3. \end{cases}$$

x	$\frac{1}{2}x-2$
-2	$-1-2=-3$
-4	$-2-2=-4$

x	$-2x+1$
-2	5
3	-5



5. (14pts) A paving company is planning a 3000-square-foot building consisting of office space and four bays for equipment, each 18 feet wide. The company wishes to minimize the construction cost, which is same as minimizing the total length of the walls.
a) Express the total length of the walls as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the building for which the total length of the walls is minimal? What is the minimal wall length?



a) $xy = 3000$ $\qquad$ $L = 4y + x + 2(x-36)$

$y = \dfrac{3000}{x}$ $\qquad$ $= 3x + 4y - 72$

$L(x) = 3x + 4 \cdot \dfrac{3000}{x} - 72 = 3x + \dfrac{12000}{x} - 72$

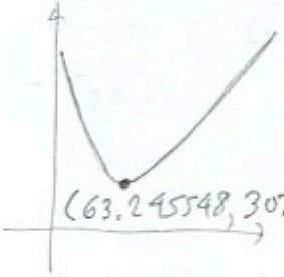
Domain:

Must have $x \geq 36$

$y > 0$

$\dfrac{3000}{x} > 0$ which is true for $x \geq 36$

Domain $[36, \infty)$



Dimensions: $\dfrac{3000}{63.245548}$

63.245548, by 47.434169

x by y

Minimal wall length $= 307.47332$