College Algebra — Joysheet 2
MAT 140, Fall 2024 — D. Ivanšić

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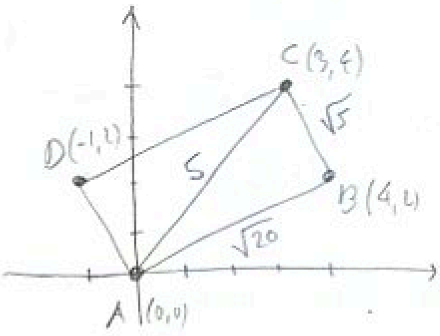
Covers: 1.1, 1.2 *Show all your work!*

1. (11pts) Draw the quadrilateral with vertices $A = (0,0)$, $B = (4,2)$, $C = (3,4)$ and $D = (-1,2)$ in the coordinate plane.
a) Compute the lengths of all sides of the quadrilateral and find its perimeter.
b) Find the length of the diagonal AC.
c) Is this quadrilateral a rectangle? (To see this, you essentially have to check that triangle ABC is a right triangle.)

$\sqrt{4.5}$



a) $d(A,B) = \sqrt{(4-0)^2+(2-0)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

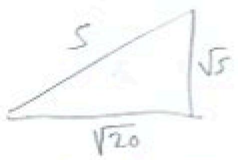
$d(B,C) = \sqrt{(3-4)^2+(4-2)^2} = \sqrt{1+4} = \sqrt{5}$

$d(C,D) = \sqrt{(-1-3)^2+(2-4)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$d(D,A) = \sqrt{(0-(-1))^2+(0-2)^2} = \sqrt{1+4} = \sqrt{5}$

Perimeter $= 2\sqrt{5}+\sqrt{5}+2\sqrt{5}+\sqrt{5} = 6\sqrt{5} \approx 13.416408$

b) $d(A,C) = \sqrt{(3-0)^2+(4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$



c) check if
$\sqrt{20}^2 + \sqrt{5}^2 \overset{?}{=} 5^2$
$20+5 \overset{?}{=} 25$

yes, so the triangle ABC is right, and the quadrilateral is a rectangle

2. (8pts) Find the equation of the circle, if its center is $(-2,-4)$ and the point $(0,-1)$ is on the circle. Draw the circle.

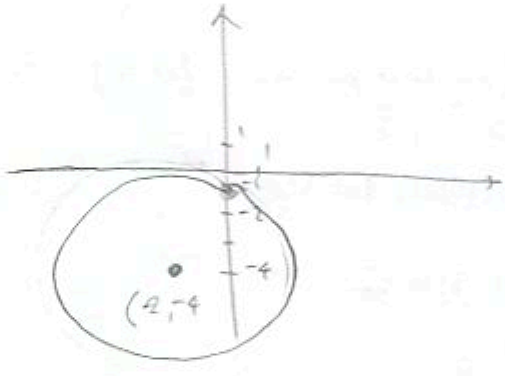
r = dist. from $(-2,-4)$ to $(0,-1)$

$= \sqrt{(0-(-2))^2+(-1-(-4))^2} = \sqrt{2^2+3^2}$

$= \sqrt{13}$

Eq. of circle: $(x-(-2))^2+(y-(-4))^2 = \sqrt{13}^2$

$(x+2)^2+(y+4)^2 = 13$



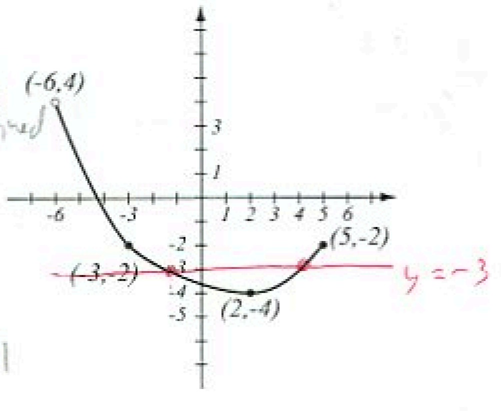
3. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(-3)$ and $f(6)$. $f(-3) = -2$, $f(6)$ not defined

b) What is the domain of f? $(-6, 5]$

c) What is the range of f? $[-4, 4)$

d) What are the solutions of the equation $f(x) = -3$? $x = -1.2, 4.1$

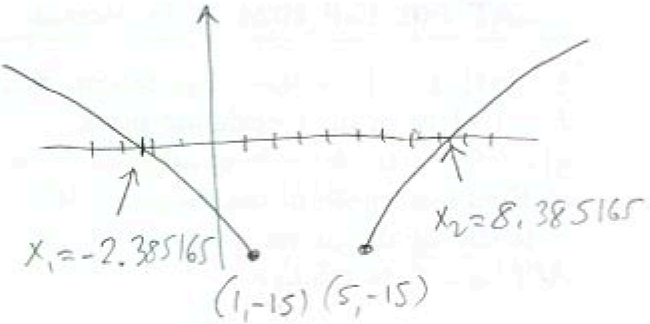


4. (12pts) The function $f(x) = 3\sqrt{x^2 - 6x + 5} - 15$ is given.

a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) State the domain and range.



b) x-int: -2.385165, 8.385165

y-int $f(0) = 3\sqrt{5} - 15 = -8.291796$

c) Domain $= (-\infty, 1] \cup [5, \infty)$

Range $= [-15, \infty)$

5. (11pts) Find the domain of each function and write it using interval notation.

$$f(x) = \sqrt{x} + \frac{4}{3x - 8}$$

Must have: $x \geq 0$ (due to $\sqrt{\ }$)

Can't have $3x - 8 = 0$

$3x = 8$

$x = \frac{8}{3}$

0 $\frac{8}{3}$

$[0, \frac{8}{3}) \cup (\frac{8}{3}, \infty)$

$$g(x) = \frac{x + 2}{x^2 - 3x + 2}$$

Can't have $x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

1 2

$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

6. (10pts) Let $h(x) = \dfrac{x^2 + x - 9}{2x - 8}$. Find the following (simplify where appropriate).

$h(-1) = \dfrac{(-1)^2 + (-1) - 9}{2(-1) - 8} = \dfrac{-9}{-10} = \dfrac{9}{10}$

$h(4) = \dfrac{4^2 + 4 - 9}{2 \cdot 4 - 8} = \dfrac{11}{0}$ not defined

$h(b^3) = \dfrac{(b^3)^2 + b^3 - 9}{2b^3 - 8} = \dfrac{b^6 + b^3 - 9}{2b^3 - 8}$

$h(u + 3) = \dfrac{(u+3)^2 + (u+3) - 9}{2(u+3) - 8}$

$= \dfrac{u^2 + 6u + 9 + u + 3 - 9}{2u + 6 - 8} = \dfrac{u^2 + 7u + 3}{2u - 2}$