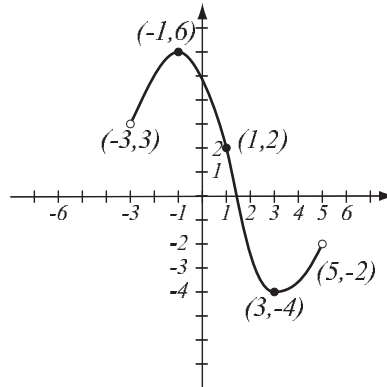


1. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- a) Find:  $f(-1) =$        $f(5) =$
- b) What is the domain of  $f$ ?
- c) What is the range of  $f$ ?
- d) What are the solutions of the equation  $f(x) = -3$ ?



2. (12pts) Use your calculator to accurately sketch the graph of  $f(x) = -x^2 + 2x + 4$ .

- a) Draw the graph on paper and indicate units on the axes.
- b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).
- c) State the range of the function in interval notation.

3. (6pts) Find the equation of the line (in form  $y = mx + b$ ) that passes through the points  $(-2, 1)$  and  $(1, -2)$ . Draw the requested line.

4. (9pts) Find the equation of the line (in form  $y = mx + b$ ) that is parallel to the line  $3x + 2y = 8$  and contains the point  $(1, -6)$ . Draw both lines.

5. (7pts) In a coordinate system, draw the triangle with vertices  $A = (-1, 1)$ ,  $B = (3, 0)$ , and  $C = (0, 5)$ .

a) Compute the slopes of the sides.

b) Use slopes to determine if this is a right triangle.

6. (10pts) Let  $f(x) = \frac{1}{x^2 - 7x + 10}$ . Find the following (simplify where appropriate).

$$f(1) =$$

$$f(2) =$$

$$f(a^3) =$$

$$f(t - 2) =$$

7. (5pts) Find the domain of the function below and write it using interval notation.

$$f(x) = \frac{4}{5 - 3x} - \frac{7}{3x + 4}$$

8. (7pts) Solve and write the solution in interval notation.

$$3 - x \geq 4 \text{ or } 5 - 2x < 1$$

9. (10pts) The endpoints of a diameter of a circle are  $(1, -3)$  and  $(5, 1)$ .

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

10. (12pts) A water company offers two plans to pay for water:

A) \$25 flat fee plus \$3 per cubic meter of water.

B) \$35 flat fee that includes 2 cubic meters, and then \$2.50 per cubic meter for usage beyond 2 cubic meters.

Assuming a customer always uses at least 2 cubic meters of water per month, for which amount of water usage is plan A better? Solve as an inequality.

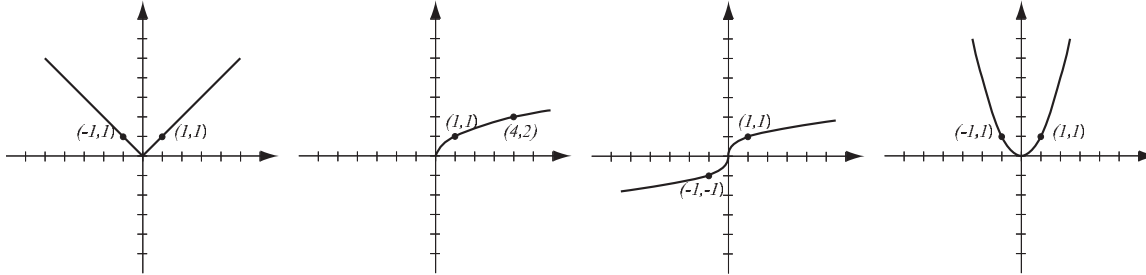
**11.** (14pts) Mike drives from Paducah to Owensboro in 2 hours. On the same road, Steve drives from Owensboro to Paducah 7mph slower than Mike, so it takes him 2 and a quarter hours.

a) How fast does Mike drive?

b) What is the distance from Paducah to Owensboro along this road?

**Bonus** (10pts) Maria has a total of \$3500 invested in two accounts, one bearing 3% and the other 4% interest. The account bearing 4% gives \$12 more in interest in one year than the account bearing 3%. How much is invested in each account?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let  $f(x) = \frac{x^2 + 1}{2x - 1}$ ,  $g(x) = \sqrt{x + 3}$ .

Find the following (simplify where possible):

$$(f + g)(1) =$$

$$(fg)(3) =$$

$$\frac{g}{f}(x) =$$

$$(g \circ f)(2) =$$

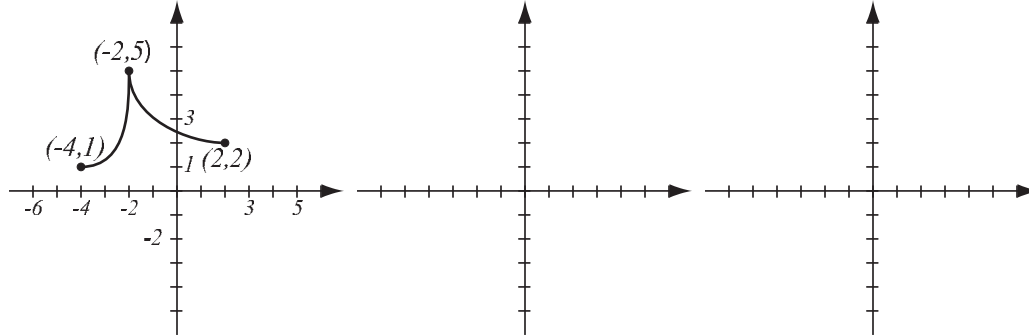
$$(f \circ g)(x) =$$

The domain of  $f + g$  in interval notation

3. (6pts) Consider the function  $h(x) = \sqrt[3]{2|x| + 1}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:  
 a) shape of  $y = x^3$ , shifted to the left by 4.  
 b) shape of  $y = \sqrt{x}$ , reflected over the  $y$ -axis and then stretched vertically by factor 5.

5. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $f(x - 1) - 2$  and  $-f(2x)$  and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x + 5, & \text{if } -3 < x \leq 1 \\ 1 - 2x, & \text{if } x > 1 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} 2x - 3, & \text{if } x < 0 \\ \sqrt{x+3}, & \text{if } 0 \leq x \leq 4 \\ \frac{x^2}{x+1}, & \text{if } 4 < x \leq 10 \end{cases}$$

$$f(4) =$$

$$f(-2) =$$

$$f(2 \cdot 4) =$$

$$f(12) =$$

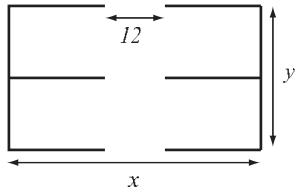
8. (20pts) Let  $f(x) = |x^2 - 10|$  (answer with 6 decimal points accuracy).

- Use your graphing calculator to accurately draw the graph of  $f$  (on paper!). Indicate units on the axes.
- Determine algebraically whether the function is odd, even, or neither.
- Verify your conclusion from b) by stating symmetry.
- Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
- State the intervals where the function is increasing and where it is decreasing.

9. (14pts) A horse breeder wishes to build a stable that is to have area 1600 square feet and four stalls with a 12-ft passageway going through the middle. To minimize cost, the total length of walls has to be as small as possible.

a) Express the total length of walls of the stable as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the stable that has the smallest total wall length? What is the smallest total wall length?



**Bonus.** (10pts) In general, every function  $f(x)$  can be written as a sum of an even and an odd function. We verify this on an example. Let  $f(x) = \frac{1}{4+x}$ .

a) Compute  $g(x) = \frac{1}{2}(f(x) + f(-x))$  and  $h(x) = \frac{1}{2}(f(x) - f(-x))$ ; don't simplify.

b) Verify algebraically that one of  $g$  and  $h$  is even and the other is odd.

c) Verify that  $g(x) + h(x) = f(x)$ .



College Algebra — Exam 3  
MAT 140C, Fall 2024 — D. Ivanić

Name: \_\_\_\_\_  
*Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $3i(4 - 5i) + 2i(-1 + 4i) =$

2. (6pts)  $\frac{3 + i}{1 - 2i} =$

3. (4pts) Simplify and justify your answer.

$i^{86} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x = -5$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|2x - 7| > 3$

6. (6pts) Let  $P(x)$  be a polynomial of degree 4.

a) Draw a graph of  $P$  that has the maximal number of  $x$ -intercepts.

b) Explain why the graph of  $P$  has to have at least one turning point.

**7.** (12pts) The quadratic function  $f(x) = x^2 - 4x + 5$  is given. Do the following without using the calculator.

a) Find the  $x$ - and  $y$ -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

Solve the equations:

**8.** (8pts)  $\frac{x}{x+1} + \frac{16}{x^2 - 6x - 7} = \frac{2}{x-7}$

**9.** (8pts)  $x + \sqrt{22 - x} = 2$

**10.** (14pts) The polynomial  $f(x) = (x + 3)(x - 1)^2(x - 4)^2$  is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the  $y$ -intercept.

c) Use the graphing calculator along with a) and b) to accurately sketch the graph of  $f$  (yes, on paper!).

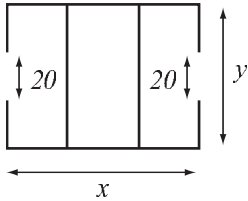
d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

**11.** (12pts) In a right triangle, the longer side is twice the length of the shorter side, and the hypotenuse is 4 centimeters longer than the shorter side. What is the length of the shorter side in this right triangle?

**12.** (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 600 feet of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



**Bonus.** (10pts) Find all solutions to the equation.

$$|x^2 - 10x + 23| = 2$$

College Algebra — Exam 4  
MAT 140C, Fall 2024 — D. Ivanić

Name: \_\_\_\_\_  
*Show all your work!*

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$$\log_4 64 = \quad \log_2 \frac{1}{8} = \quad \log_a \sqrt[7]{a^6} = \quad \log_{a^2} a^8 =$$

2. (4pts) Use the change-of-base formula and your calculator to find  $\log_{12} 6$  with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If  $\log_a 3 = 0.6826$  and  $\log_a 7 = 1.2091$ , calculate the following values:

$$\log_a \frac{7}{3} = \quad \log_a 63 =$$

4. (4pts) Simplify.

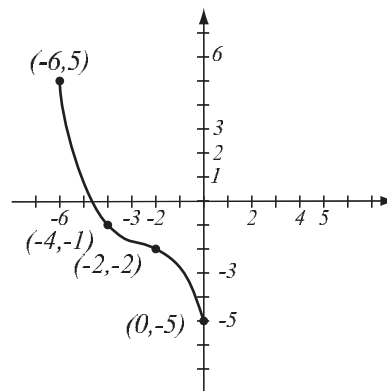
$$\log 10^{|x|} = \quad 6^{\log_6(x^2-7)} =$$

5. (8pts) If you deposit \$2,000 in an account bearing 4.2% interest, compounded monthly, how much is in the account after 3 years?

6. (6pts) The graph of a function  $f$  is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points, and showing any asymptotes.



7. (9pts) Let  $f(x) = \frac{2x + 3}{x}$ .

a) Find the formula for  $f^{-1}$ .

b) Find the range of  $f^{-1}$ .

8. (6pts) Using transformations, draw the graph of  $f(x) = -\log_2(x + 3)$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Indicate at least one point on the graph and any asymptotes.

**9.** (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \left( 81x^5 \sqrt[3]{y^4} \right) =$$

$$\ln \frac{x^2 y^4}{e^4 x^{\frac{3}{2}}} =$$

**10.** (12pts) Write as a single logarithm. Simplify if possible.

$$3 \log(a^4 b^2) + 2 \log(a^3 b^{-3}) =$$

$$3 \log_7(3x^2 y^4) + 4 \log_7 \sqrt{y} - 2 \log_7(6x^{-2} y^5) =$$

Solve the equations.

**11.** (6pts)  $3^{4x-2} = \left(\frac{1}{9}\right)^{x-3}$

**12.** (8pts)  $e^{2x-3} = 5^{4x-7}$

**13.** (12pts) Census data has the population of Elizabethtown, KY, as 28,500 in 2010 and 31,400 in 2020. Assume that it has grown according to the formula  $P(t) = P_0e^{kt}$ .

a) Find  $k$  and write the function that describes the population at time  $t$  years since 2010. Graph it on paper.

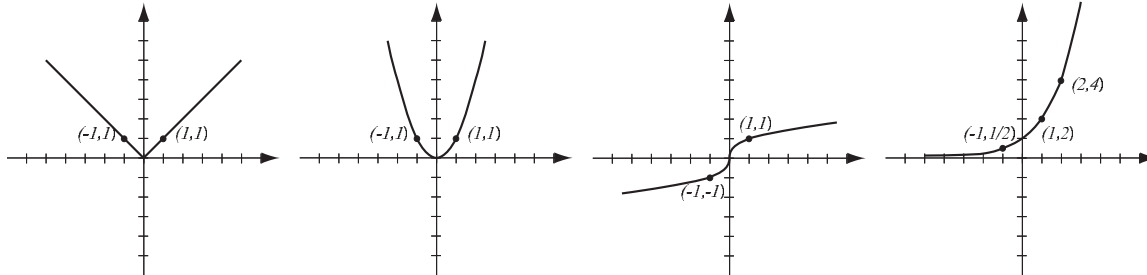
b) Find the predicted population in the year 2028.

**Bonus** (10pts) Solve the equation.

$$\log_2(x - 3) + \log_2(x + 1) = 5$$

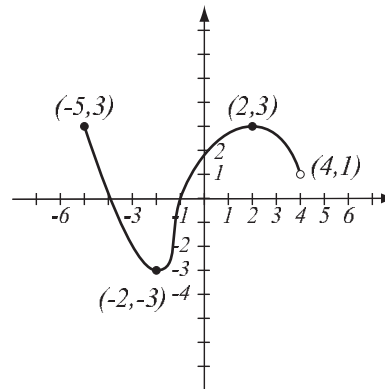


1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- Find:  $f(-2) =$        $f(5) =$
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the solutions of the equation  $f(x) = -2$ ?



3. (10pts)

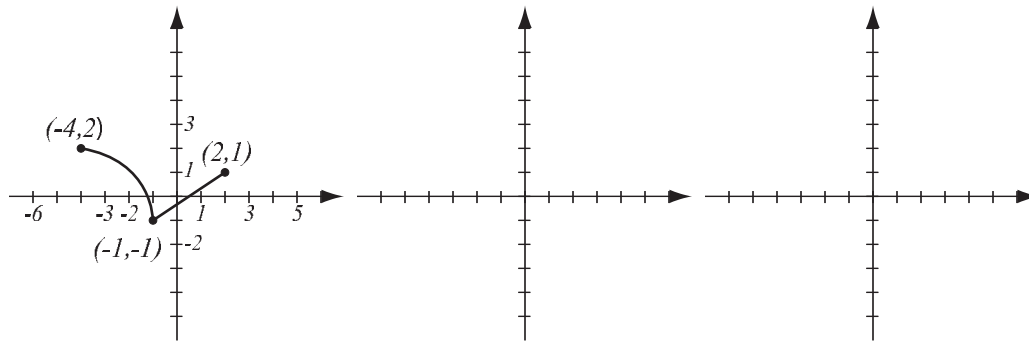
- Find the equation of the line that passes through points  $(1, -1)$  and  $(4, 5)$ .
- Find the equation of the line (in form  $y = mx + b$ ) that is parallel to the line in a) and passes through the point  $(-2, 0)$ .
- Draw both lines.

4. (3pts) Find the domain of the function  $f(x) = \sqrt{2x - 5}$  and write it in interval notation.

5. (6pts) Solve and write the solution in interval notation.

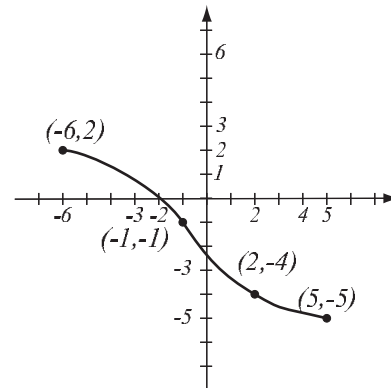
$$|x + 8| \leq 2$$

6. (10pts) The graph of  $f(x)$  is drawn below. Find the graphs of  $f(-x) + 2$  and  $2f(x + 1)$  and label all the relevant points.



7. (6pts) The graph of a function  $f$  is given.

- Is this function one-to-one? Justify.
- If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



**8.** (12pts) The quadratic function  $f(x) = x^2 + 4x + 7$  is given. Do the following without using the calculator.

a) Find the  $x$ - and  $y$ -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

**9.** (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_5 (125x^7 \sqrt[3]{y}) =$$

**10.** (6pts) Write as a single logarithm. Simplify if possible.

$$2 \log(x^4 y^2) - 4 \log(x^3 y) =$$

11. (8pts) Let  $f(x) = \frac{x}{x^2 - 5}$ ,  $g(x) = \sqrt{x - 3}$ . Find the following (simplify where possible):

$$\frac{f}{g}(x) =$$

$$(f \circ g)(x) =$$

12. (20pts) The polynomial  $P(x) = x^3 - 4x$  is given (answer with 6 decimals accuracy).

- What is the end behavior of the polynomial?
- Factor the polynomial to find all the zeros and their multiplicities. Find the  $y$ -intercept.
- Determine algebraically whether the function is odd, even, or neither.
- Use the graphing calculator along with a) and b) to sketch the graph of  $P$  (yes, on paper!).
- Verify your conclusion from c) by stating symmetry.
- Find all the turning points (i.e., local maxima and minima).

13. (8pts) Solve the equation.

$$\frac{2x}{x+4} + \frac{10x-8}{x^2+2x-8} = \frac{x}{x-2}$$

**14.** (14pts) Mike drives from Paducah to Owensboro in 2 hours. On the same road, Steve drives from Owensboro to Paducah 13mph slower than Mike, so it takes him 2 and a half hours.

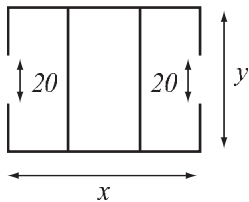
a) How fast does Mike drive?

b) What is the distance from Paducah to Owensboro along this road?

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**16.** (12pts) Census data has the population of Elizabethtown, KY, as 28,500 in 2010 and 31,400 in 2020. Assume that it has grown according to the formula  $P(t) = P_0e^{kt}$ .

a) Find  $k$  and write the function that describes the population at time  $t$  years since 2010. Graph it on paper.

b) Find the predicted population in the year 2030.

**Bonus** (10pts) Find all solutions to the equation.

$$|x^2 - 10x + 23| = 2$$