College Algebra — Final Exam
MAT 140C, Fall 2024 — D. Ivanšić

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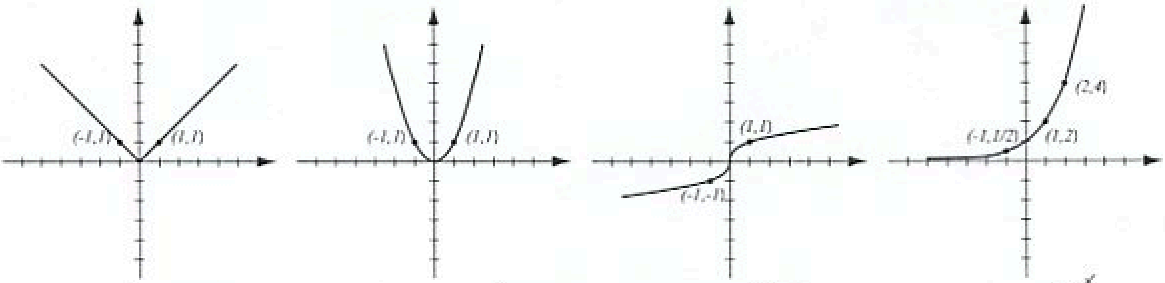
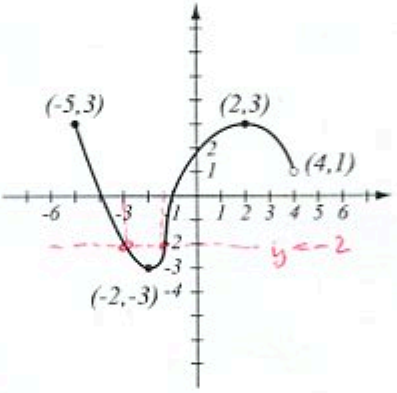
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=|x|$ $\quad$ $y=x^2$ $\quad$ $y=\sqrt[3]{x}$ $\quad$ $y=2^x$

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-2)=-3$ $\quad f(5)=$ not defined

b) What is the domain of f? $[-5,4)$

c) What is the range of f? $[-3,3]$

d) What are the solutions of the equation $f(x)=-2$?

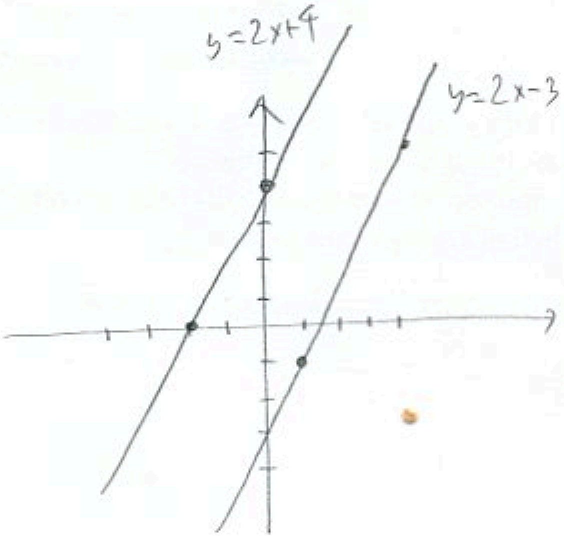
$x=-3,-1.4$

3. (10pts)

a) Find the equation of the line that passes through points $(1,-1)$ and $(4,5)$.

b) Find the equation of the line (in form $y=mx+b$) that is parallel to the line in a) and and passes through the point $(-2,0)$.

c) Draw both lines.



a) $m=\dfrac{5-(-1)}{4-1}=\dfrac{6}{3}=2$

$y-(-1)=2(x-1)$

$y+1=2x-2$

$y=2x-3$

b) slope is 2, through $(-2,0)$

$y-0=2(x-(-2))$

$y=2x+4$

4. (3pts) Find the domain of the function $f(x) = \sqrt{2x-5}$ and write it in interval notation.

Must have $2x - 5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$[\frac{5}{2}, \infty)$

5. (6pts) Solve and write the solution in interval notation.

$|x+8| \leq 2$

$-2 \leq x + 8 \leq 2$

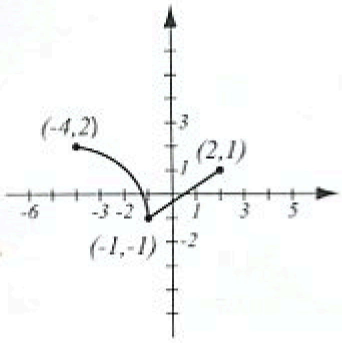
$-2-8 \leq x \leq 2-8$

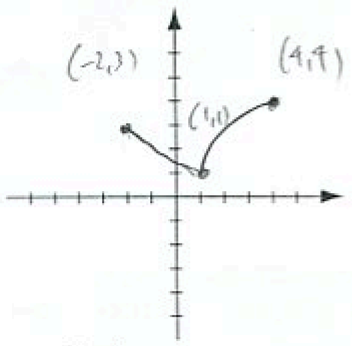
$-10 \leq x \leq -6$

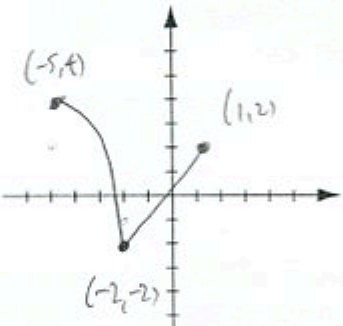
$-10 \quad -6$

$[-10, -6]$

6. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x)+2$ and $2f(x+1)$ and label all the relevant points.



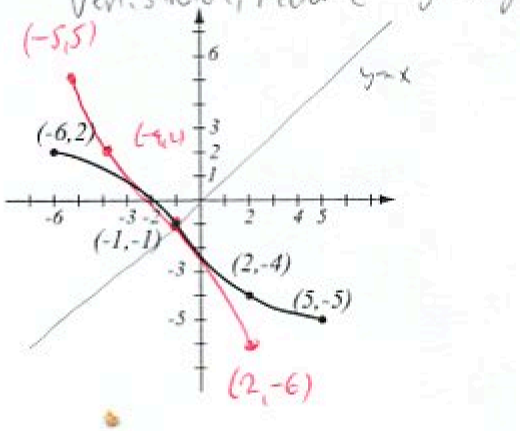




reflect in y-axis $x \mapsto -x$
shift up 2 $y \mapsto y+2$

shift left 1 $x \mapsto x-1$
vert. stretch, Factor 2 $y \mapsto 2y$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes, it passes horizontal line test

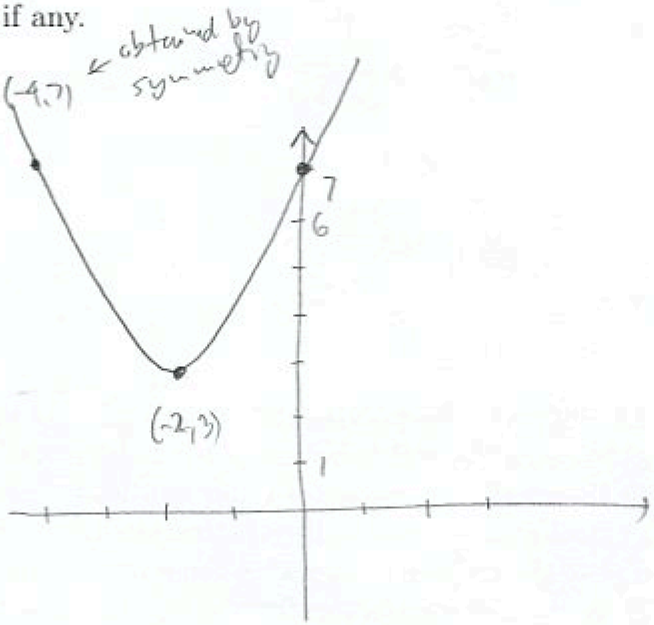
b) picture

8. (12pts) The quadratic function $f(x) = x^2 + 4x + 7$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) y-int: $f(0) = 7$

x-int: $x^2 + 4x + 7 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4.1.7}}{2 \cdot 1} = \frac{-4 \pm \sqrt{16-28}}{2}$$

$$= \frac{-4 \pm \sqrt{-12}}{2}$$ no real sol, so no x-int.

b) $h = -\frac{b}{2a} = -\frac{4}{2.1} = -2$

$k = f(-2) = (-2)^2 + 4(-2) + 7 = 4 - 8 + 7 = 3$

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_5 \left(125x^7 \sqrt[3]{y}\right) =$ $\log_5 125 + \log_5 x^7 + \log_5 y^{\frac{1}{3}}$

$= 3 + 7\log_5 x + \frac{1}{3}\log_5 y$

10. (6pts) Write as a single logarithm. Simplify if possible.

$2\log(x^4y^2) - 4\log(x^3y) =$ $\log(x^4y^2)^2 - \log(x^3y)^4$

$= \log(x^8y^4) - \log(x^{12}y^4)$

$= \log \frac{x^8y^4}{x^{12}y^4} = \log \frac{1}{x^4}$

11. (8pts) Let $f(x) = \dfrac{x}{x^2-5}$, $g(x) = \sqrt{x-3}$. Find the following (simplify where possible):

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{x}{x^2-5}}{\sqrt{x-3}} = \dfrac{x}{x^2-5} \cdot \dfrac{1}{\sqrt{x-3}}$

$= \dfrac{x}{(x^2-5)\sqrt{x-3}}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x-3})$

$= \dfrac{\sqrt{x-3}}{\sqrt{x-3}^{\,2}-5} = \dfrac{\sqrt{x-3}}{x-3-5} = \dfrac{\sqrt{x-3}}{x-8}$

12. (20pts) The polynomial $P(x) = x^3 - 4x$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) like x^3

b) $x^3-4x = x(x^2-4) = x(x-2)(x+2)$

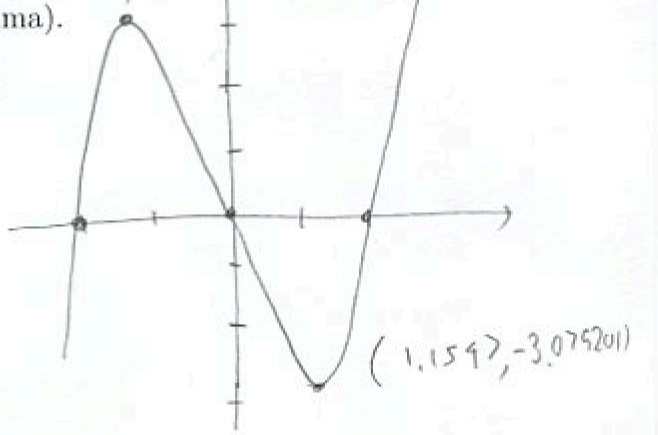
zero	0	2	−2
mult	1	1	1

y-int: $P(0)=0$

c) $P(-x) = (-x)^3 - 4(-x) = -x^3+4x = -P(x)$

odd function

d)



e) symm wrt origin

f) local max is $3.079201 = P(-1.1547)$
local min is $-3.079201 = P(1.1547)$

13. (8pts) Solve the equation.

$$\frac{2x}{x+4} + \frac{10x-8}{x^2+2x-8} = \frac{x}{x-2}$$

x^2+2x-8: $(x+4)(x-2)$ $1 \cdot (x+4)(x-2)$

$\dfrac{2x}{\cancel{x+4}} \cdot (\cancel{x+4})(x-2) + \dfrac{10x-8}{\cancel{(x+4)}\cancel{(x-2)}}(\cancel{x+4})(\cancel{x-2}) = \dfrac{x}{\cancel{x-2}}(x+4)(\cancel{x-2})$

$2x(x-2) + 10x - 8 = x(x+4)$

$2x^2 - 4x + 10x - 8 = x^2 + 4x$ $|-x^2-4x$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4, 2$

but both give a 0 in denominator, so

$\boxed{\text{no solution}}$

14. (14pts) Mike drives from Paducah to Owensboro in 2 hours. On the same road, Steve drives from Owensboro to Paducah 13mph slower than Mike, so it takes him 2 and a half hours.
a) How fast does Mike drive?
b) What is the distance from Paducah to Owensboro along this road?

$$P \underset{d,\ r-13,\ 2.5}{\overset{d,\ r,\ 2}{\rightleftarrows}} O$$

a) Mike drives 65mph

b) $d = 65 \cdot 2 = 130$ miles

$d = r \cdot 2$

equal → $d = (r-13) \cdot 2.5$

$2r = 2.5(r-13)$

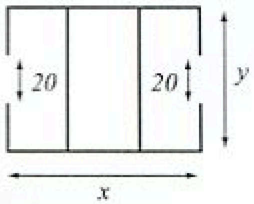
$2r = 2.5r - 32.5$

$-0.5r = -32.5$

$0.5r = 32.5$

$r = \frac{32.5}{0.5} = 65$ mph

15. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 800 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



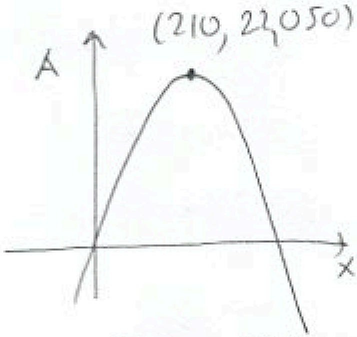
$A = xy = x(210 - \frac{1}{2}x) = -\frac{1}{2}x^2 + 210x$

$2x + y - 20 + y + y + y - 20 = 800$

$2x + 4y - 40 = 800$

$4y = 840 - 2x$

$y = 210 - \frac{1}{2}x$



Domain: must have

$x \geq 0$

$y \geq 20$

$210 - \frac{1}{2}x \geq 20$

$-\frac{1}{2}x \geq -190$

$x \leq 380$

Domain: $[0, 380]$

$h = -\frac{210}{2 \cdot (-\frac{1}{2})} = \frac{210}{1} = 210$

$k = 210(210 - 105) = 210 \cdot 105 = 22{,}050$

dimensions: 210 by 105

max area: 22050 sq ft

16. (12pts) Census data has the population of Elizabethtown, KY, as 28,500 in 2010 and 31,400 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
b) Find the predicted population in the year 2030.

a) $P(t) = 285e^{kt}$ (in hundreds)

$314 = 285e^{k\cdot 10}$

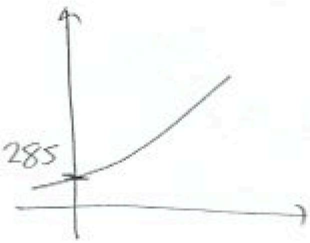
ln | $\frac{314}{285} = e^{10k}$

$\ln\frac{314}{285} = \ln e^{10k}$

$\ln\frac{314}{285} = 10k$

$k = \frac{\ln\frac{314}{285}}{10} = 0.00969038$

$P(t) = e^{0.00969038t}$



b) $P(20) = 285e^{0.00969038\cdot 20} =$

$= 345.95...$

About 34,595 inhabitants

Bonus (10pts) Find all solutions to the equation.

$|x^2 - 10x + 23| = 2$

$x^2 - 10x + 23 = 2$ or $x^2 - 10x + 23 = -2$

$x^2 - 10x + 21 = 2$ $\quad$ $x^2 - 10x + 25 = 0$

$(x-7)(x-3) = 0$ $\quad$ $(x-5)^2 = 0$

$x = 3, 7$ $\quad$ $x = 5$

Solutions: 3, 5, 7