1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_4 64 = 3$ $\qquad$ $\log_2 \frac{1}{8} = -3$ $\qquad$ $\log_a \sqrt[7]{a^6} = \frac{6}{7}$ $\qquad$ $\log_{a^2} a^8 = 4$

$4^? = 64$ $\qquad$ $2^? = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$ $\qquad$ $a^? = \sqrt[7]{a^6} = a^{\frac{6}{7}}$ $\qquad$ $(a^2)^? = a^8$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_{12} 6$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_{12} 6 = \dfrac{\ln 6}{\ln 12} = 0.721057$

3. (5pts) If $\log_a 3 = 0.6826$ and $\log_a 7 = 1.2091$, calculate the following values:

$\log_a \frac{7}{3} = \log_a 7 - \log_a 3$
$= 1.2091 - 0.6826$
$= 0.5265$

$\log_a 63 = \log_a(9 \cdot 7) = \log_a(3^2 \cdot 7)$
$= 2\log_a 3 + \log_a 7$
$= 2 \cdot 0.6826 + 1.2091 = 2.5743$

4. (4pts) Simplify.

$\log 10^{|x|} = |x|$ $\qquad\qquad$ $6^{\log_6(x^2-7)} = x^2 - 7$

5. (8pts) If you deposit \$2,000 in an account bearing 4.2% interest, compounded monthly, how much is in the account after 3 years?

$A = P\left(1 + \frac{r}{n}\right)^{nt} = 2000\left(1 + \frac{0.042}{12}\right)^{12 \cdot 3} = 2000 \cdot 1.134\ldots = 2268.07$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.
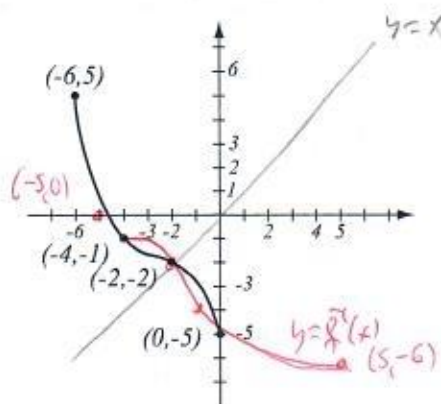
a) yes, it passes horizontal line test

b)

(-6,5)

$y=x$

(-5,0)

(-4,-1)

(-2,-2)

(0,-5)

$y=f^{-1}(x)$

(5,-6)

7. (9pts) Let $f(x) = \dfrac{2x+3}{x}$.
a) Find the formula for f^{-1}.
b) Find the range of f^{-1}.

$y = \dfrac{2x+3}{x}$ $| \cdot x$
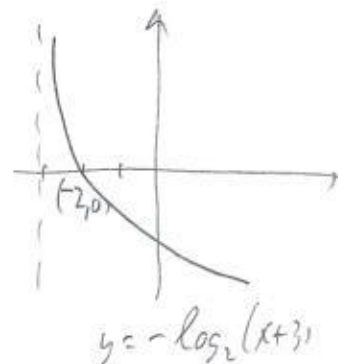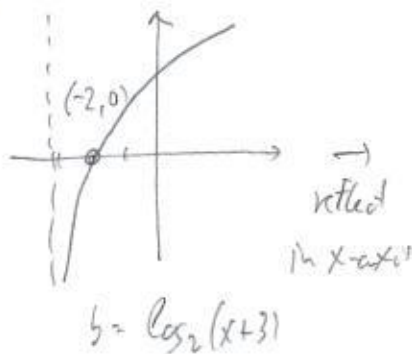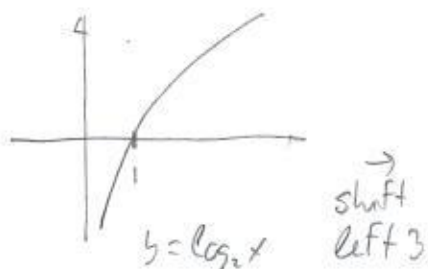
$xy = 2x+3$

$xy - 2x = 3$

$x(y-2) = 3$

$x = \dfrac{3}{y-2}$

$f^{-1}(y) = \dfrac{3}{y-2}$

range of $f^{-1} =$ domain of $f = (-\infty, 0) \cup (0, \infty)$

Can't have $x=0$

8. (6pts) Using transformations, draw the graph of $f(x) = -\log_2(x+3)$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

$y = \log_2 x$ shift left 3

$y = \log_2(x+3)$

(-2,0)

reflect in x-axis

$y = -\log_2(x+3)$

(-2,0)

9. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \left(81x^5 \sqrt[3]{y^4}\right) = \log_3 81 + \log_3 x^5 + \log_3 y^{\frac{4}{3}}$

$\qquad\qquad = 4 + 5\log_3 x + \frac{4}{3}\log_3 y$

$\ln \dfrac{x^2 y^4}{e^4 x^{\frac{3}{2}}} = \ln x^2 + \ln y^4 - \ln e^4 - \ln x^{\frac{3}{2}}$

$\qquad\qquad = 2\ln x + 4\ln y - 4 - \frac{3}{2}\ln x = \frac{1}{2}\ln x + 4\ln y - 4$

10. (12pts) Write as a single logarithm. Simplify if possible.

$3\log(a^4 b^2) + 2\log(a^3 b^{-3}) = \log\left(a^4 b^2\right)^3 + \log\left(a^3 b^{-3}\right)^2 = \log\left(a^{12} b^6\right) + \log\left(a^6 b^{-6}\right)$

$\qquad\qquad = \log\left(a^{12} b^6 \, a^6 b^{-6}\right) = \log a^{18}$

$3\log_7(3x^2 y^4) + 4\log_7 \sqrt{y} - 2\log_7(6x^{-2}y^5) = \log_7\left(3x^2 y^4\right)^3 + \log_7 \sqrt{y}^{\,4} - \log_7\left(6x^{-2}y^5\right)^2$

$= \log_7\left(27 x^6 y^{12}\right) + \log_7 y^2 - \log_7\left(36 x^{-4} y^{10}\right) = \log_7 \dfrac{27 x^6 y^{12} \cdot y^2}{36 x^{-4} y^{10}} = \log_7 \dfrac{3 x^{6-(-4)} y^{12+2-10}}{4}$

$= \log_7 \dfrac{3 x^{10} y^4}{4}$

Solve the equations.

11. (6pts) $3^{4x-2} = \left(\dfrac{1}{9}\right)^{x-3}$

$3^{4x-2} = \left(3^{-2}\right)^{x-3}$

$3^{4x-2} = 3^{-2x+6}$

$4x - 2 = -2x + 6$

$6x = 8$

$x = \dfrac{8}{6} = \dfrac{4}{3}$

12. (8pts) $e^{2x-3} = 5^{4x-7}$ | ln

$\ln e^{2x-3} = \ln 5^{4x-7}$

$2x - 3 = (4x-7)\ln 5$

$2x - 3 = 4x\ln 5 - 7\ln 5$ | $-4x\ln 5 + 3$

$2x - 4x\ln 5 = 3 - 7\ln 5$

$x(2 - 4\ln 5) = 3 - 7\ln 5$

$x = \dfrac{3 - 7\ln 5}{2 - 4\ln 5} = \dfrac{7\ln 5 - 3}{4\ln 5 - 2} = 1.86267$

13. (12pts) Census data has the population of Elizabethtown, KY, as 28,500 in 2010 and 31,400 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
b) Find the predicted population in the year 2028.
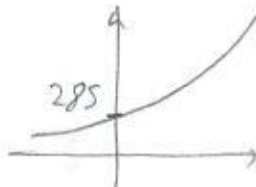
$P = P_0 e^{kt}$

$P(t) = 285 e^{kt}$ (in hundreds)

$314 = 285 e^{k \cdot 10}$

$\dfrac{314}{285} = e^{10k} \quad | \, \ln$

$\ln \dfrac{314}{285} = 10k$

$k = \dfrac{\ln \frac{314}{285}}{10} = 0.00969038$

285

b) $t = 18$

$P(18) = 285 e^{0.00969038 \cdot 18}$

$= 339.31$

About 33,931 inhabitants in 2028

Bonus (10pts) Solve the equation.

$\log_2(x - 3) + \log_2(x + 1) = 5$

$\log_2 \big((x-3)(x+1)\big) = 5$

$2^{\log_2((x-3)(x+1))} = 2^5$

$(x-3)(x+1) = 32$

$x^2 - 2x - 3 = 32$

$x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x = 7, -5$

Since $-5 - 3 < 0$, -5 gives a neg. in $\log_2(x-3)$ so is not a solution

$x = 7$ is a solution