College Algebra — Exam 3
MAT 140C, Fall 2024 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $3i(4-5i)+2i(-1+4i) =$ $12i - 15i^2 - 2i + 8i^2$

$= 10i + 15 - 8 = 7 + 10i$

2. (6pts) $\frac{3+i}{1-2i} = \frac{3+i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{3+6i+i+2i^2}{1^2-(2i)^2} = \frac{3+7i-2}{1-(-4)} = \frac{1+7i}{5}$

3. (4pts) Simplify and justify your answer.

$i^{86} =$ $i^{84} \cdot i^2 = \underbrace{(i^4)}_{=1}{}^{21} \cdot i^2 = 1 \cdot (-1) = -1$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x = -5$ $\quad 1 + 5^2$

$x^2 - 2 \cdot x \cdot 5 + 5^2 = -5 + 5^2$

$(x-5)^2 = 20$

$x - 5 = \pm\sqrt{20}$

$x = 5 \pm \sqrt{20} = 5 \pm 2\sqrt{5}$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|2x - 7| > 3$

$2x - 7 > 3$ or $2x - 7 < -3$

$2x > 10 \qquad 2x < 4$

$x > 5$ or $x < 2$

2 5

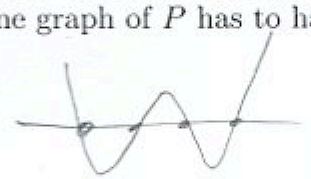
$(-\infty, 2) \cup (5, \infty)$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.

a) Draw a graph of P that has the maximal number of x-intercepts.

b) Explain why the graph of P has to have at least one turning point.

a) 4 x-int



b) because overall shape is $\cup$ or $\cap$ at some point between left and right ends the curve will have to turn.

7. (12pts) The quadratic function $f(x) = x^2 - 4x + 5$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

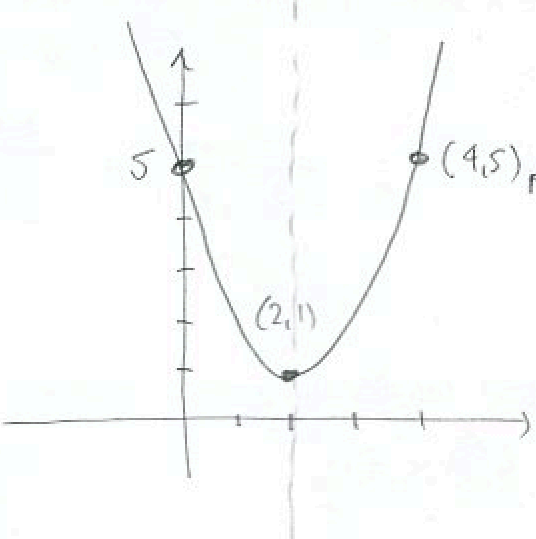
a) y-int: $f(0)=5$

x-int: $x^2-4x+5=0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

no real sol., so no x-int.



inferred by symm. over line $x=2$

b). vertex: $h = -\frac{b}{2a} = -\frac{-4}{2 \cdot 1} = 2$ $k = f(2) = 2^2 - 4 \cdot 2 + 5 = 1$

Solve the equations:

8. (8pts) $\dfrac{x}{x+1} + \dfrac{16}{x^2-6x-7} = \dfrac{2}{x-7}$ $\Big| \cdot (x+1)(x-7)$

$(x+1)(x-7)$

$$\frac{x}{\cancel{x+1}}\cancel{(x+1)}(x-7) + \frac{16}{\cancel{(x+1)}\cancel{(x-7)}}\cancel{(x+1)}\cancel{(x-7)} = \frac{2}{\cancel{x-7}}(x+1)\cancel{(x-7)}$$

$x(x-7) + 16 = 2(x+1)$

$x^2 - 7x + 16 = 2x + 2$ $\big| -2x-2$

$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = 2, \cancel{7}$ gives 0 in a denom

only $x=2$ is a sol.

9. (8pts) $x + \sqrt{22-x} = 2$ $\big| -x$

$\sqrt{22-x} = 2-x$ $\big|^2$

$22 - x = 4 - 2 \cdot 2 \cdot x + x^2$ $\big| +x-22$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x = 6, -3$

Check: $6 + \sqrt{22-6} \overset{?}{=} 2$

$6 + \sqrt{16} = 2$ no

$-3 + \sqrt{22-(-3)} = 2$

$-3 + \sqrt{25} = 2$ yes

$x=-3$ is the solution

10. (14pts) The polynomial $f(x) = (x+3)(x-1)^2(x-4)^2$ is given.

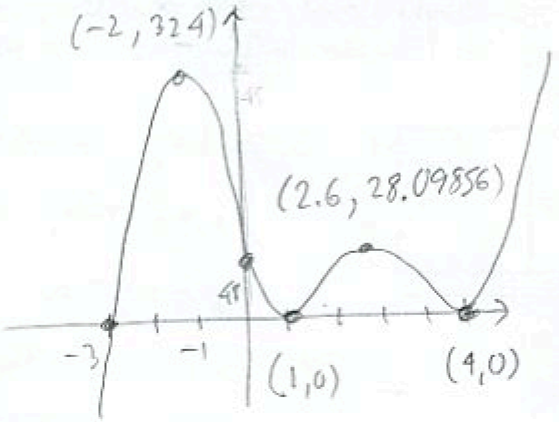
a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y-intercept.

c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).

d) Find all the turning points (i.e., local maxima and minima) with accuracy 6 decimal points.

a) $(x)(x)^2(x)^2 = x^5$, like x^5



b)

zeros	-3	1	4
mult	1	2	2

$f(0) = 3 \cdot (-1)^2 (-4)^2 = 48$

d) Turning points.

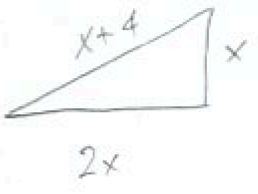
(-2, 324)

(1, 0)

(2.6, 28.09856)

(4, 0)

11. (12pts) In a right triangle, the longer side is twice the length of the shorter side, and the hypothenuse is 4 centimeters longer than the shorter side. What is the length of the shorter side in this right triangle?



$x^2 + (2x)^2 = (x+4)^2$

$x^2 + 4x^2 = x^2 + 2 \cdot x \cdot 4 + 4^2$

$5x^2 = x^2 + 8x + 16 \quad | -x^2 - 8x - 16$

$4x^2 - 8x - 16 = 0 \quad | \div 4$

$x^2 - 2x - 4 = 0$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \dfrac{2 \pm \sqrt{20}}{2} = \dfrac{2 \pm 2\sqrt{5}}{2} = \dfrac{2(1 \pm \sqrt{5})}{2} = 1 \pm \sqrt{5}$

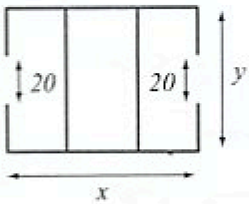
4+16

Since $1 - \sqrt{5} < 0$ and $x > 0$, only $1 + \sqrt{5}$ is the solution

12. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 600 feet of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



$A = xy = x\left(-\frac{1}{2}x + 160\right) = -\frac{1}{2}x^2 + 160x$

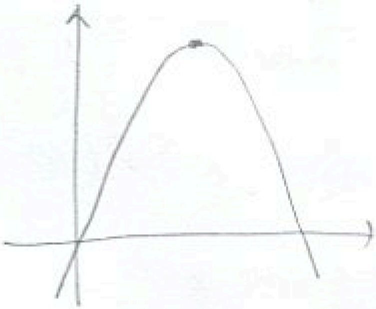
$2x + 2y + 2(y-20) = 600$

$2x + 4y - 40 = 600$

$2x + 4y = 640$

$4y = -2x + 640 \quad | \div 4$

$y = -\frac{1}{2}x + 160$



Domain:

Must have:

$x \geq 0$

$y \geq 20$

$-\frac{1}{2}x + 160 \geq 20$

$-\frac{1}{2}x \geq -140 \quad |(-2)$

$x \leq 280$

Domain $[0, 280]$

$h = -\frac{b}{2a} = -\frac{160}{2\left(-\frac{1}{2}\right)} = 160$

$k = -\frac{1}{2}160^2 + 160 \cdot 160 = \frac{1}{2}160^2 = 12800$

Dimensions: 160 by 80 $\leftarrow -\frac{1}{2} \cdot 160 + 160 = y$

Max area: 12 800 ft^2

Bonus. (10pts) Find all solutions to the equation.

$|x^2 - 10x + 23| = 2$

$x^2 - 10x + 23 = \pm 2$

$x^2 - 10x + 23 = 2$

$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x = 3, 7$

$x^2 - 10x + 23 = -2$

$x^2 - 10x + 25 = 0$

$(x-5)(x-5) = 0$

$x = 5$