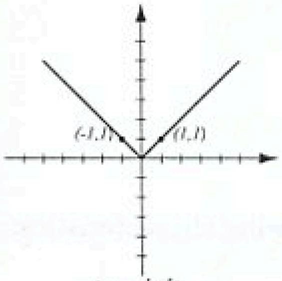
College Algebra — Exam 2 MAT 140C, Fall 2024 — D. Ivanšić

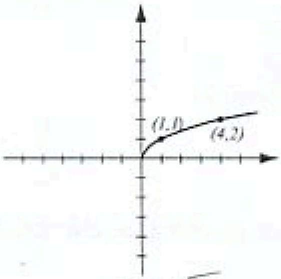
Name: Saul Ocean

Show all your work!

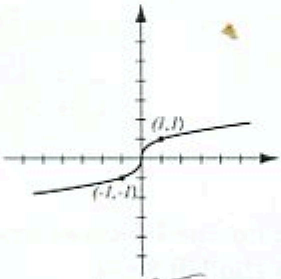
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



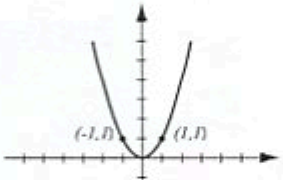
$y = |x|$



$y = \sqrt{x}$



$y = \sqrt[3]{x}$



$y = x^2$

2. (20pts) Let $f(x) = \dfrac{x^2+1}{2x-1}$, $g(x) = \sqrt{x+3}$.

Find the following (simplify where possible):

$(f+g)(1) = f(1) + g(1) = \dfrac{1^2+1}{2\cdot 1-1} + \sqrt{1+3}$

$= \dfrac{2}{1} + \sqrt{4} = 4$

$(fg)(3) = f(3)\cdot g(3) = \dfrac{3^2+1}{2\cdot 3-1}\cdot\sqrt{3+3}$

$= \dfrac{10}{5}\cdot\sqrt{6} = 2\sqrt{6}$

$\dfrac{g}{f}(x) = \dfrac{\sqrt{x+3}}{\frac{x^2+1}{2x-1}} = \dfrac{\sqrt{x+3}}{1}\cdot\dfrac{2x-1}{x^2+1}$

$= \dfrac{(2x-1)\sqrt{x+3}}{x^2+1}$

$(g\circ f)(2) = g(f(2)) = g\left(\dfrac{2^2+1}{2\cdot 2-1}\right)$

$= g\left(\dfrac{5}{3}\right) = \sqrt{\dfrac{5}{3}+3} = \sqrt{\dfrac{14}{3}}$

$(f\circ g)(x) = f(g(x)) = f(\sqrt{x+3}) = \dfrac{\sqrt{x+3}^2+1}{2\sqrt{x+3}-1} = \dfrac{x+3+1}{--} = \dfrac{x+4}{2\sqrt{x+3}-1}$

The domain of $f+g$ in interval notation

domain f: can't have $2x-1=0$

$2x=1$

$x=\frac{1}{2}$

domain g: must have $x+3 \geq 0$

$x \geq -3$

Domain f+g: $[-3, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

3. (6pts) Consider the function $h(x) = \sqrt[3]{2|x|+1}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = 2|x|+1$ $\qquad\qquad$ $g(x) = |x|$

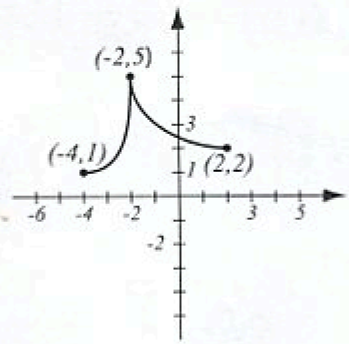
$f(x) = \sqrt[3]{x}$ $\qquad\qquad$ $f(x) = \sqrt[3]{2x+1}$

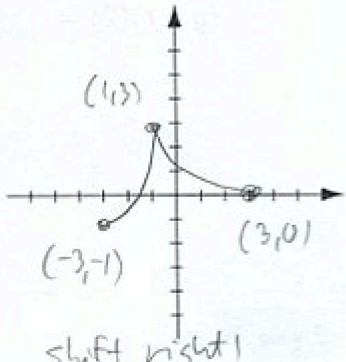
4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^3$, shifted to the left by 4.
b) shape of $y = \sqrt{x}$, reflected over the y-axis and then stretched vertically by factor 5.

a) $y = (x+4)^3$ $\qquad$ b) $\sqrt{x} \rightsquigarrow \sqrt{-x} \rightsquigarrow 5\sqrt{-x}$

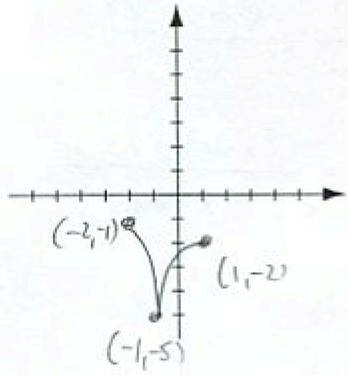
$y = 5\sqrt{-x}$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x-1)-2$ and $-f(2x)$ and label all the relevant points.





shift right 1
down 2



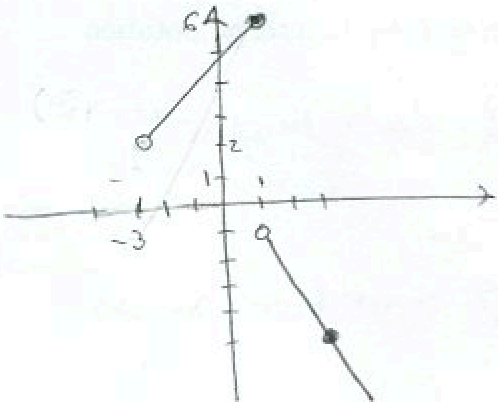
horiz. stretch
factor $\frac{1}{2}$
reflect in x-axis

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} x+5, & \text{if } -3 < x \leq 1 \\ 1-2x, & \text{if } x > 1 \end{cases}$$

x	x+5
-3	2
1	6

x	1-2x
1	-1
3	-5



7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} 2x-3, & \text{if } x < 0 \\ \sqrt{x+3}, & \text{if } 0 \le x \le 4 \\ \dfrac{x^2}{x+1}, & \text{if } 4 < x \le 10 \end{cases}$$

$f(4) =$ $\sqrt{4+3} = \sqrt{7}$

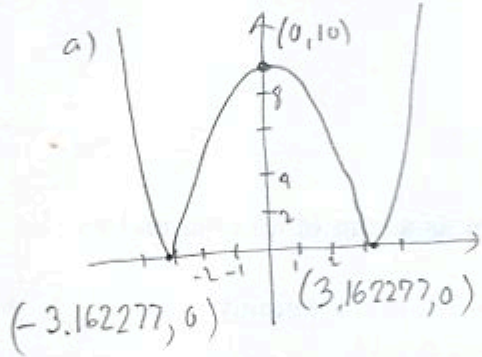
$f(-2) =$ $2(-2)-3 = -7$

$f(2 \cdot 4) =$ $f(8) = \dfrac{8^2}{8+1} = \dfrac{64}{9}$

$f(12) =$ not defined

8. (20pts) Let $f(x) = |x^2 - 10|$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

a)



d) local max: $10 = f(0)$

local min: $0 = f(-3.162277)$

$0 = f(3.162277)$

e) Increasing on

$(-3.162277, 0)$ and $(3.162277, \infty)$

Decreasing on

$(-\infty, -3.162277)$ and $(0, 3.162277)$

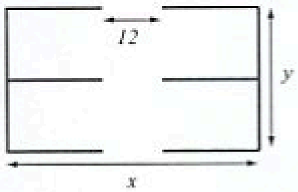
b) $f(-x) = |(-x)^2 - 10|$

$= |x^2 - 10| = f(x)$

Function is even

c) graph is symm wrt y-axis

9. (14pts) A horse breeder wishes to build a stable that is to have area 1600 square feet and four stalls with a 12-ft passageway going through the middle. To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the stable as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the stable that has the smallest total wall length? What is the smallest total wall length?



$A = 1600 = xy \Rightarrow y = \frac{1600}{x}$

$L = 3(x-12) + 2y = 3x + 2y - 36 = 3x + 2\cdot\frac{1600}{x} - 36$

$L(x) = 3x + \frac{3200}{x} - 36$

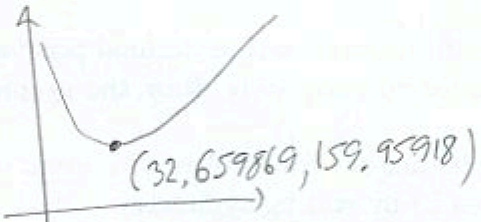
Dimensions:

32.659869 by 48.989786

Minimal wall length:

159.95918

Domain:
must have $x \geq 12$
no upper bound
$[12, \infty)$



Bonus. (10pts) In general, every function $f(x)$ can be written as a sum of an even and an odd function. We verify this on an example. Let $f(x) = \frac{1}{4+x}$.
a) Compute $g(x) = \frac{1}{2}(f(x) + f(-x))$ and $h(x) = \frac{1}{2}(f(x) - f(-x))$; don't simplify.
b) Verify algebraically that one of g and h is even and the other is odd.
c) Verify that $g(x) + h(x) = f(x)$.

a) $g(x) = \frac{1}{2}\left(\frac{1}{4+x} + \frac{1}{4-x}\right) \quad h(x) = \frac{1}{2}\left(\frac{1}{4+x} - \frac{1}{4-x}\right)$

b) $g(-x) = \frac{1}{2}\left(\frac{1}{4-x} + \frac{1}{4-(-x)}\right) = \frac{1}{2}\left(\frac{1}{4-x} + \frac{1}{4+x}\right) = g(x)$ so g is even

$h(-x) = \frac{1}{2}\left(\frac{1}{4-x} - \frac{1}{4-(-x)}\right) = \frac{1}{2}\left(\frac{1}{4-x} - \frac{1}{4+x}\right) = -\frac{1}{2}\left(\frac{1}{4+x} - \frac{1}{4-x}\right) = -h(x)$ so h is odd

c) $\frac{1}{2}\left(\frac{1}{4+x} + \frac{1}{4-x}\right) + \frac{1}{2}\left(\frac{1}{4+x} - \frac{1}{4-x}\right) = \frac{1}{2}\left(\frac{1}{4+x} + \cancel{\frac{1}{4-x}} + \frac{1}{4+x} - \cancel{\frac{1}{4-x}}\right) = \frac{1}{2}\cdot 2\cdot\frac{1}{4+x} = \frac{1}{4+x}$