College Algebra — Exam 1
MAT 140C, Fall 2024 — D. Ivanšić

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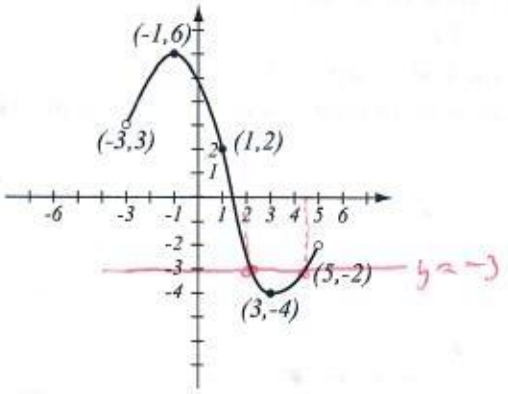
Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



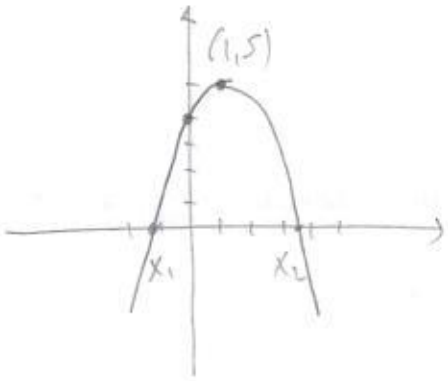
a) Find: $f(-1) = 6$ $\quad f(5) =$ not def.

b) What is the domain of f? $(-3, 5)$

c) What is the range of f? $[-4, 6]$

d) What are the solutions of the equation $f(x) = -3$? $x = 2, 4.5$

2. (12pts) Use your calculator to accurately sketch the graph of $f(x) = -x^2 + 2x + 4$.



a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

c) State the range of the function in interval notation.

b) y-int: $f(0) = 4$ $\quad$ Range: $(-\infty, 5]$

x-int: $X_1 = -1.236068$

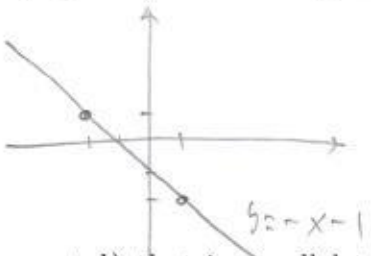
$X_2 = 3.236068$

3. (6pts) Find the equation of the line (in form $y = mx + b$) that passes through the points $(-2, 1)$ and $(1, -2)$. Draw the requested line.

$m = \frac{-2-1}{1-(-2)} = \frac{-3}{3} = -1$ $\quad y - 1 = -1(x-(-2))$

$y - 1 = -x - 2$

$y = -x - 1$



4. (9pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $3x + 2y = 8$ and contains the point $(1, -6)$. Draw both lines.

$3x + 2y = 8$

$2y = -3x + 8 \quad | \div 2$

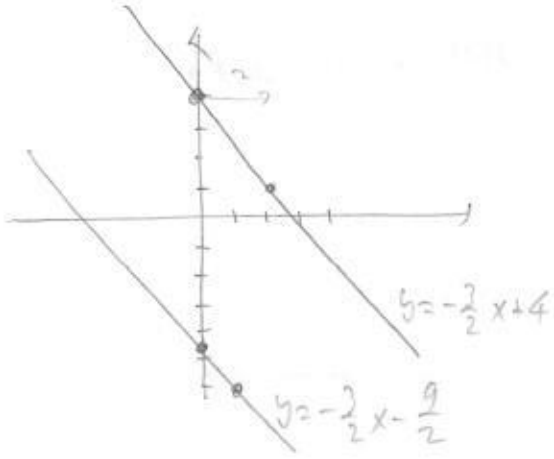
$y = -\frac{3}{2}x + 4$

parallel line has same slope, $-\frac{3}{2}$

$y - (-6) = -\frac{3}{2}(x - 1)$

$y + 6 = -\frac{3}{2}x + \frac{3}{2}$

$y = -\frac{3}{2}x - \frac{9}{2}$

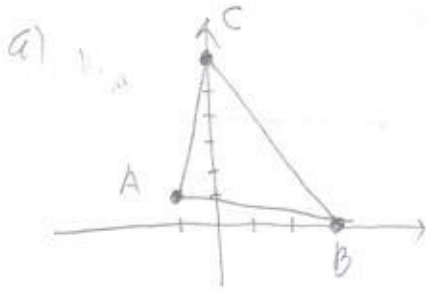


5. (7pts) In a coordinate system, draw the triangle with vertices $A = (-1, 1)$, $B = (3, 0)$, and $C = (0, 5)$.

a) Compute the slopes of the sides.

b) Use slopes to determine if this is a right triangle.

a)



$m_{AB} = \frac{0-1}{3-(-1)} = -\frac{1}{4}$

$m_{BC} = \frac{5-0}{0-3} = -\frac{5}{3}$

$m_{AC} = \frac{5-1}{0-(-1)} = \frac{4}{1} = 4$

AB and AC are perpendicular, because their slopes, $-\frac{1}{4}$ and 4 are opposite reciprocal

Triangle is a right triangle

6. (10pts) Let $f(x) = \dfrac{1}{x^2 - 7x + 10}$. Find the following (simplify where appropriate).

$f(1) = \dfrac{1}{1^2 - 7\cdot 1 + 10} = \dfrac{1}{4}$

$f(2) = \dfrac{1}{2^2 - 7\cdot 2 + 10} = \dfrac{1}{4 - 14 + 10} = \dfrac{1}{0}$ not defined

$f(a^3) = \dfrac{1}{(a^3)^2 - 7a^3 + 10} = \dfrac{1}{a^6 - 7a^3 + 10}$

$f(t-2) = \dfrac{1}{(t-2)^2 - 7(t-2) + 10}$

$= \dfrac{1}{t^2 - 4t + 4 - 7t + 14 + 10} = \dfrac{1}{t^2 - 11t + 28}$

7. (5pts) Find the domain of the function below and write it using interval notation.

$f(x) = \dfrac{4}{5 - 3x} - \dfrac{7}{3x + 4}$

Can't have:

$5 - 3x = 0$ $\quad 3x + 4 = 0$

$5 = 3x$ $\quad 3x = -4$

$x = \frac{5}{3}$ $\quad x = -\frac{4}{3}$

Domain is

$(-\infty, -\frac{4}{3}) \cup (\frac{4}{5}, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$

11. (14pts) Mike drives from Paducah to Owensboro in 2 hours. On the same road, Steve drives from Owensboro to Paducah 7mph slower than Mike, so it takes him 2 and a quarter hours.

a) How fast does Mike drive?

b) What is the distance from Paducah to Owensboro along this road?

a) Mike drives 63 mph

b) $d = 2 \cdot 63 = 126$ miles

$d = 2r$

$d = 2.25(r-7)$

$2r = 2.25(r-7)$

$2r = 2.25r - 15.75 \quad | -2r$

$0 = 0.25r - 15.75 \quad | +15.75$

$15.75 = 0.25r$

$r = \frac{15.75}{0.25} = 63$

Bonus (10pts) Maria has a total of \$3500 invested in two accounts, one bearing 3% and the other 4% interest. The account bearing 4% gives \$12 more in interest in one year than the account bearing 3%. How much is invested in each account?

x = amt invested at 3%

$3500 - x$ = ——— at 4%

Interest From 3% + 12 = interest from 4%

$0.03 \cdot x \cdot 1 + 12 = 0.04(3500-x) \cdot 1$

$0.03x + 12 = 140 - 0.04x \quad | +0.04x$

$0.07x + 12 = 140$

$0.07x = 128$

$x = \frac{128}{0.07} = 1828.57$ invested at 3%

$3500 - 1828.57 = 1671.43$ invested at 4%

8. (7pts) Solve and write the solution in interval notation.

$3 - x \geq 4$ or $5 - 2x < 1$

$-x \geq 1$ $\quad -2x < -4$

$x \leq -1$ or $\quad x > \frac{-4}{-2}$

$x > 2$

$(-\infty, -1] \cup (2, \infty)$

9. (10pts) The endpoints of a diameter of a circle are $(1, -3)$ and $(5, 1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

Center $= \left(\frac{1+5}{2}, \frac{-3+1}{2}\right) = (3, -1)$

radius = dist. from $(3,-1)$ to $(1,-3)$

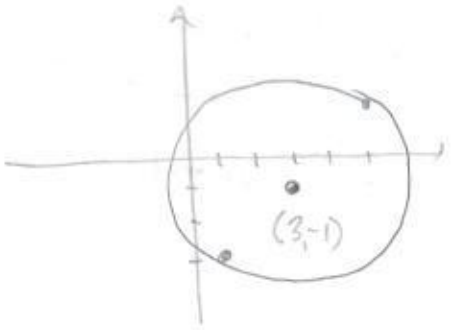
$= \sqrt{(1-3)^2 + (-3-(-1))^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{8}$

Eq. of circle:

$(x-1)^2 + (y-(-1))^2 = \sqrt{8}^2$

$(x-1)^2 + (y+1)^2 = 8$



10. (12pts) A water company offers two plans to pay for water:
A) \$25 flat fee plus \$3 per cubic meter of water.
B) \$35 flat fee that includes 2 cubic meters, and then \$2.50 per cubic meter for usage beyond 2 cubic meters.
Assuming a customer always uses at least 2 cubic meters of water per month, for which amount of water usage is plan A better? Solve as an inequality.

x = cubic meters of water used

plan A: $25 + 3x$

plan B: $35 + 2.5(x-2)$

Asking when is

$25 + 3x \leq 35 + 2.5(x-2)$

$25 + 3x \leq 35 + 2.5x - 5$ $\quad 1-25$

$3x \leq 5 + 2.5x$ $\quad 1-2.5x$

$0.5x \leq 5$

$x \leq \frac{5}{0.5}$

$x \leq 10$

For usage under 10 cubic meters, plan A is cheaper.