

1. (6pts) Let $\mathbf{u} = \langle 3, 1, -7 \rangle$ and $\mathbf{v} = \langle 2, -3, 1 \rangle$. Find the angle between \mathbf{u} and \mathbf{v} .

2. (16pts) The paraboloids $z = \frac{1}{2}(x^2 + y^2)$ and $z = 36 - x^2 - y^2$ intersect in a circle.

a) Sketch a picture.

b) Find a parametrization for the circle.

c) Find the parametric equation of the tangent line to the circle at point $(3\sqrt{2}, -\sqrt{6}, 12)$ and draw it on your sketch.

3. (10pts) A line is given parametrically: $x = 1 + 2t$, $y = -3 - t$, $z = 5 + 2t$. Find the equation of the plane that contains this line and the point $(-3, 7, 0)$.

4. (12pts) Let $f(x, y) = x^2 - y$.

a) Draw a rough contour map for the function with contour interval 1, going from $c = -3$ to $c = 3$.

b) Find ∇f and roughly draw this vector field. Note that no computation is needed to draw the vector field.

c) What is $\int_C \nabla f \cdot ds$ if C is the arc of the unit circle that is in the first quadrant, going counterclockwise?

5. (16pts) Let $f(x, y) = ye^{xy}$, $x = u^3 - v^3$, $y = \frac{u}{v}$. Find $\frac{\partial f}{\partial v}$ when $u = 2$, $v = 1$.

6. (16pts) Find and classify the local extremes for the function $f(x, y) = x^3 - xy + y^3$.

7. (18pts) Find $\iint_D x \, dA$ if D is the region bounded by the curves $y = x^2 - 10$ and $y = 5x + 14$.

8. (16pts) Use cylindrical or spherical coordinates to set up $\iiint_W \frac{x^2 + y^2 + z^2}{x^2 + y^2 + 1} \, dV$ where W is the region inside the sphere $x^2 + y^2 + z^2 \leq 16$, between the planes $y = \sqrt{3}x$ and $y = -\frac{1}{\sqrt{3}}x$, and where $y \geq 0$. Sketch the region of integration. Do not evaluate the integral.

9. (24pts) Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if S is the part of the cone $z = 2\sqrt{x^2 + y^2}$ for which $z \leq 12$, and $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$. Use the normal vectors to the surface that point upwards. Draw the surface and some normal vectors, parametrize the surface and specify the planar region D where your parameters come from.

10. (16pts) Sketch the region W given by $x^2 + y^2 \leq z \leq 25$, $x \geq y$, $x, y \geq 0$. Then write the two iterated triple integrals that stand for $\iiint_W f \, dV$ which end in $dz \, dx \, dy$, and $dx \, dz \, dy$.

Bonus. (15pts) A spherical cap of height h is the set $x^2 + y^2 + z^2 \leq R^2$, $z \geq R - h$. Show that its surface area is $A = 2\pi R h$. Then use this formula to get the surface area of a ball of radius R .