1. (19pts) Let $\phi(x, y) = \sqrt{x^2 + y^2}$.
a) Find $\nabla\phi(x, y)$. What is $||\nabla\phi(x, y)||$?
b) Roughly draw the vector field $\nabla\phi(x, y)$, scaling the vectors for a better picture.
c) How could you have roughly done b) without the actual computation in a)?
d) What is $\int_C \nabla\phi \cdot d\mathbf{s}$ if C is part of the curve $y = x^3$ from $(0, 0)$ to $(2, 8)$? How about if C is a straight line segment from $(0, 0)$ to $(2, 8)$?

2. (15pts) Let C be part of the helix $x = 4\cos t$, $y = 4\sin t$, $z = t$, for $t \in [2\pi, 4\pi]$.
a) Set up $\int_C xz\, ds$.
b) Set up $\int_C \mathbf{F} \cdot d\mathbf{s}$, if $\mathbf{F}(x, y, z) = \langle y, x, z^2 \rangle$.
In both cases simplify the set-up, but do not evaluate the integral.

3. (16pts) One of the two vectors fields below is not a gradient field, and the other one is (cross partials, remember?). Identify which is which, and find the potential function for the one that is.

$\mathbf{F}(x, y, z) = \langle x^2 + y^2, y^2 + z^2, z^2 + x^2 \rangle$ $\qquad$ $\mathbf{G}(x, y, z) = \langle e^z, e^z, e^z(x + y + z + 1) \rangle$

4. (24pts) Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, if S is the part of the cylinder $x^2 + y^2 = 9$ between the planes $z = 1$ and $z = 5$, and $\mathbf{F}(x, y, z) = \langle -y, x - z, y \rangle$. (The surface does not include the top or the bottom, just part of the cylinder.) Use the normal vectors to the surface that point toward the z-axis. Draw the surface and some normal vectors, parametrize the surface and specify the planar region D where your parameters come from.

5. (26pts) Find the surface integral $\iint_S xz\,dS$, if S is part of the plane $4x + 2y + z = 4$ that is in the octant $x, y, z \geq 0$. Draw the surface (intercepts with the axes will help you draw the plane), parametrize it and specify the planar region D where your parameters come from.

Bonus. (10pts) A spherical cap (eek! It again!) of height h is the set $x^2 + y^2 + z^2 \leq R^2$, $z \geq R - h$. Show that its surface area is $A = 2\pi Rh$. Then use this formula to get the surface area of a ball or radius R.