Spring '10/MAT 309/Exam 3 **Name:** *Show all your work.*

1. (12pts) Find the equation of the tangent plane to the surface $x^2 - \frac{y^2}{4} + z^2 = 1$ at the point $\left(1, \sqrt{2}, \frac{\sqrt{2}}{2}\right)$. Simplify the equation to standard form.

2. (20pts) A bug is moving along the path $\mathbf{r}(t) = \langle 2t+3, t^2 \rangle$ through a region where temperature is distributed according to the function $T(x, y) = \dfrac{e^{x-y}}{x}$ (in °C).

a) Find the point P where the bug is at $t = 3$.

b) At what rate is the bug's temperature changing when $t = 3$ (in seconds)? What are the units?

c) At P, in which direction does the temperature decrease the fastest?

3. (20pts) Let $f(x,y) = x\ln(x^2+y^2)$, $x = \sin u + \cos v$, $y = \cos u \sin v$. Find $\dfrac{\partial f}{\partial u}$ when $u = \pi$, $v = \frac{\pi}{2}$.

4. (12pts) A cylinder is measured to have radius $x = 20$cm and height $y = 12$cm, with an error in measurement at most 0.75cm in each. Estimate the maximal error in computing the volume of the cylinder.

5. (12pts) Find $\dfrac{\partial y}{\partial z}$ using implicit differentiation, if $x^2y + y^3z + z^4x = 13$.

6. (24pts) Find and classify the local extremes for the function $f(x, y) = x^2y^2 + y^4 - x^2 + 6y$.

Bonus (10pts) Consider the function $f(x, y) = x + \sqrt{3}y$ on the domain $x^2 + y^2 \leq 1$.
a) Determine the global minimum and maximum values of f.
b) Draw the graph of f and justify your answer from a) using the picture.