Fall '10/MAT 309/Exam 2 **Name:** *Show all your work.*

1. (10pts) Write the parametrization of the circle that is the intersection of the sphere $x^2 + y^2 + z^2 = 16$ with the plane $x = 2$. Sketch a picture.

2. (20pts) A curve is given by $\mathbf{r}(t) = \langle 4t, t\cos t, t\sin t\rangle$, $t \in [0, 4\pi]$.
a) Sketch this curve.
b) Find the parametric equation of the tangent line to the curve at time $t = \pi$ and draw this tangent line on your sketch.

3. (22pts) After another ill-fated attempt at lunch, Wile E. Coyote finds himself ejected from the edge of a 60-meter tall canyon at angle 30° above the horizontal with initial speed 40 meters per second.
a) Find his position at time t. (For simplicity of calculation, blaspheme away and set $g = 10$.)
b) When does he hit the bottom of the canyon?
c) What is his speed when he hits the bottom?

4. (18pts) Find the length of the curve with the parametrization $\mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{2\sqrt{2}}{\sqrt{3}} t^{\frac{3}{2}}, 3t + 7 \right\rangle$, $t \in [1, 5]$.

5. (20pts) Let $f(x, y) = x^2y$.
a) Identify and draw vertical traces for this function.
b) Using a), draw the graph of the function (in your 3-D coordinate system, orient the x-axis to the right, and the y-axis away from you).
c) Draw a rough contour map for the function, with contour interval 1, going from $c = -3$ to $c = 3$.
d) By looking at the contour map, indicate the direction (if any), in which we would have to move from $(1, 2)$ in order to decrease the value of the function.

6. (10pts) Determine and sketch the domain of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 9}$.

Bonus (10pts) Let $\mathbf{r}(t)$ the position of a moving object in space. If $\mathbf{r}'''(t) = \mathbf{0}$, use differentiation rules for products to help you show that the volume of the parallelepiped spanned by the position, velocity and acceleration vectors is constant. *(Hint: triple product.)*