Advanced Calculus 2 Test Knowledge — Spring 2010

Sections 8.1–8.4

Definitions

Pointwise convergence of a sequence of functions (8.1.1)
Uniform convergence of a sequence of functions (8.1.4)
Uniform norm of a bounded function (8.1.7)
Definition and properties of the logarithmic function (8.3.8, 8.3.9)
Definition of e (8.3.5)
Definition of the general power function (8.3.10)
Definition of the general logarithmic function (8.3.14)
Definition of π (8.4.10)

Theorems

Lemma 8.1.5
Lemma 8.1.8
Cauchy Criterion for Uniform Convergence (8.1.10)
Interchange of Limit and Continuity (8.2.2)
Interchange of Limit and Derivative (8.2.3)
Interchange of Limit and Integral (8.2.4)
Existence of the exponential function (8.3.1)
Properties of the exponential function ((i)–(vi) in 8.3.1, 8.3.6 and 8.3.7)
Corollary 8.3.3
Uniqueness of the exponential function (8.3.4)
Existence of cos and sin (8.4.1)
Properties of cos and sin ((i)–(vi) in 8.4.1, 8.4.2, 8.4.3, 8.4.6)
Uniqueness of cos and sin (8.4.4)

Proofs

Lemma 8.1.5
Cauchy Criterion for Uniform Convergence (8.1.10)
Interchange of Limit and Continuity (8.2.2)
Interchange of Limit and Integral (8.2.4)
Existence of the exponential function (8.3.1)
Corollary 8.3.3
Uniqueness of the exponential function (8.3.4)

Sections 7.1–7.4

Definitions
- Tagged partititions (7.1)
- Riemann Integral (7.1.1)
- Null set (7.3.10)
- Indefinite integral (7.3.3)
- Left, Right, Trapezoidal, Midpoint, Simpson approximations (7.4)

Theorems
- Theorem 7.1.4
- Theorem 7.1.5
- Cauchy Criterion (7.2.1)
- Squeeze Theorem (7.2.3)
- Theorem 7.2.6
- Additivity Theorem (7.2.8)
- Fundamental Theorem of Calculus (First Form) (7.3.1)
- Theorem 7.3.4
- Fundamental Theorem of Calculus (Second Form) (7.3.5)
- Theorem 7.3.6
- Lebesgue's Integrability Criterion (7.3.12)
- Substitution Theorem (7.3.8)
- Corollary 7.3.15
- Integration by Parts (7.3.17)
- Taylor's Theorem with Remainder (7.3.18)
- Error estimates for Trapezoidal, Midpoint, Simpson approximations (7.4.4, 7.4.7, 7.4.8)

Proofs
- Theorem 7.1.5
- Exercise 7.1.15
- Theorem 7.2.6
- Fundamental Theorem of Calculus (First Form) (7.3.1)
- Fundamental Theorem of Calculus (Second Form) (7.3.5)
- Corollary 7.3.15
- Taylor's Theorem with Remainder (7.3.18)

Sections 5.4, 5.6, 6.1–6.4

Definitions	Uniform continuity (5.4.1) Lipschitz function (5.4.4) (Strictly) increasing/decreasing/monotone function (5.6) Derivative of a function (6.1.1) Relative minimum/maximum/extremum (6.2) Taylor polynomial at x_0 (6.4) Convex function (6.4.5)
Theorems	Uniform Continuity Theorem (5.4.3) Uniform approximation of a continuous function (5.4.10, 5.4.13, 5.4.14) Corollary 5.6.2 Continuous Inverse Theorem (5.6.5) Differentiable function is continuous (6.1.2) Theorem 6.1.3, 6.1.6, 6.1.8 Interior Extremum Theorem (6.2.1) Rolle's Theorem (6.2.3) Mean Value Theorem (6.2.4) Theorem 6.2.7 L'Hospital's Rule I, II (6.3.3, 6.3.5) Taylor's Theorem (6.4.1) Theorem 6.4.6 Newton's Method (6.4.7)
Proofs	Uniform Continuity Theorem (5.4.3) Differentiable function is continuous (6.1.2) Theorem 6.1.3 Example 6.1.7(e) Interior Extremum Theorem (6.2.1) Rolle's Theorem (6.2.3) Mean Value Theorem (6.2.4) Theorem 6.2.7

Advanced Calculus 1 Test Knowledge — Fall 2009

Sections 1.1, 1.2, 2.1–2.4

Definitions

- Injective, surjective, bijective function (1.1.9)
- ϵ-neighborhood of a (2.2.7)
- Set bounded above/below, bounded, unbounded (2.3.1)
- Supremum and infimum of a set (2.3.2)

Theorems and Axioms

- Well-ordering property of $\mathbf{N}$ (1.2.1)
- Principle of mathematical induction (1.2.2)
- Algebraic properties of $\mathbf{R}$ (2.1.1)
- Order properties of $\mathbf{R}$ (2.1.5)
- Theorem 2.1.7
- Triangle inequality (2.2.3)
- Completeness Property of $\mathbf{R}$ (2.3.6)
- Lemmas 2.3.3 and 2.3.4

Proofs

- Nonexistence of rational number r so that $r^2 = 2$ (2.1.4)
- Theorem 2.1.9
- Existence of $\sqrt{2}$ (2.4.7)
- Archimedean Property (2.4.3)
- Density of $\mathbf{Q}$ (2.4.8)

Sections 3.1–3.6

Definitions
- Limit of a sequence (3.1.3)
- Bounded sequence (3.2.1)
- Monotone sequence (3.3.1)
- Euler's number e (3.3.6)
- Subsequence of a sequence (3.4.1)
- Cauchy sequence (3.5.1)
- Contractive sequence (3.5.7)
- Sequences tending to ∞ or $-\infty$ (3.6.1)

Theorems
- Theorem 3.2.2
- Limit Theorems (3.2.3)
- Squeeze Theorem (3.2.7)
- Monotone Convergence Theorem (3.3.2)
- Monotone Subsequence Theorem (3.4.7)
- Bolzano-Weierstrass Theorem (3.4.8)
- Cauchy Convergence Criterion (3.5.5)
- Theorem 3.6.4

Proofs
- Theorem 3.2.2
- Limit Theorems (3.2.3)
- Monotone Convergence Theorem (3.3.2)
- Example 3.3.5
- Bolzano-Weierstrass Theorem (3.4.8)
- Cauchy Convergence Criterion (3.5.5)
- Theorem 3.5.8

Sections 4.1–4.3, 5.1–5.3

Definitions	Cluster point (4.1.1) Limit of a function (4.1.4) Boundedness on a neighborhood (4.2.1) Infinite limit (4.3.5) Limit at infinity (4.3.10) Continuity (5.1.1, 5.1.5) Boundedness of a function (5.3.1) Absolute maximum or minimum (5.3.3)
Theorems	Theorem 4.1.2 Sequential criteria for limits (4.1.8, 4.1.9) Limit Theorems (4.2.4) Squeeze Theorem (4.2.7) Sequential criteria for continuity (5.1.3, 5.1.4) Combinations of continuous functions (5.2.1, 5.2.2, 5.2.6, 5.2.7) Boundedness Theorem (5.3.2) Maximum-Minimum Theorem (5.3.4) Intermediate Value Theorem (5.3.7) Theorem 5.3.9
Proofs	Theorem 4.1.2 Sequential criterion (4.1.8) Limit Theorems by definition (4.2.4) Example 5.1.6(h) Composition of continuous functions (5.2.6) Boundedness Theorem (5.3.2) Maximum-Minimum Theorem (5.3.4) Location of Roots Theorem (5.3.5)