Spring '06/MAT 145/Worksheet 8 **Name:** Solution *Show all your work.*

1. (6pts) Solve the triangle: $b = 5$, $c = 8$, $\alpha = 37°$.

$a^2 = 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos 37°$

$a^2 = 25.109\ldots$

$a = 5.01$

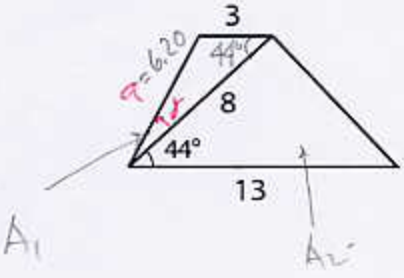
May use law of cosines for other angle, too

$\cos\beta = \dfrac{5.01^2 + 8^2 - 5^2}{2 \cdot 5.01 \cdot 8} = 0.7996\ldots$

$\beta = \arccos 0.7996\ldots = 36.91°$

$\gamma = 106.09$

2. (8pts) Consider the pictured trapezoid (top and bottom sides are parallel).
a) Find the length and angle to the horizontal of the left slanted side.
b) Find the area of the trapezoid.



a) $a^2 = 3^2 + 8^2 - 2 \cdot 3 \cdot 8 \cos 44°$

$a^2 = 38.47\ldots$

$a = 6.20$

$\cos\gamma = \dfrac{6.20^2 + 8^2 - 3^2}{2 \cdot 6.20 \cdot 8} = 0.94\ldots$

$\gamma = \arccos 0.9418\ldots$

$\gamma = 19.63$

Angle with horizontal is $44 + \gamma = 63.63$

b) Area of trapezoid

$= A_1 + A_2$

$= \frac{1}{2} 8 \cdot 3 \cdot \sin 44° + \frac{1}{2} 8 \cdot 13 \cdot \sin 44°$

$= \sin 44° (12 + 52)$

$= 64 \sin 44°$

$= 44.46$ units^2

3. (8pts) Solve the triangle: $a = 4$, $b = 7$, $c = 6$. Then find its area in two ways: by using one of the angles and by using Heron's formula. Did you get the same answer?

$$\cos\gamma = \frac{4^2+7^2-6^2}{2\cdot 4\cdot 7} = \frac{29}{56} = 0.5178$$

$$\gamma = \arccos 0.5178 = 58.81^\circ$$

$$\cos\beta = \frac{4^2+6^2-7^2}{2\cdot 4\cdot 6} = 0.0625$$

$$\beta = \arccos 0.0625 = 86.42^\circ$$

$$\alpha = 180^\circ - (58.81^\circ + 86.42^\circ) = 34.77^\circ$$

$$A = \frac{1}{2}ab\sin\gamma = \frac{1}{2}\cdot 4\cdot 7\cdot \sin 58.81^\circ = 11.98$$

$$s = \frac{1}{2}(4+7+6) = 8.5$$

$$A = \sqrt{8.5\cdot 4.5\cdot 1.5\cdot 2.5} = \sqrt{143.4375} = 11.98$$

4. (8pts) In attempting to fly from Chicago to Louisville, a distance of 330 miles, a pilot took a course that was 10° in error.

a) If the aircraft averaged 220mph and if the error is discovered after 15 minutes, through what angle should the pilot turn to head toward Louisville?

b) What new average speed should the pilot maintain so that the total time of the trip is 90 minutes?

a) $s = \text{speed}\cdot\text{time} = 220\ \text{mph}\cdot\frac{1}{4}\ \text{hr} = 55\ \text{mi}$

$$b^2 = 55^2+330^2-2\cdot 55\cdot 330\cos 10^\circ = 76176.47$$

$$b = 276.0\ \text{mi}$$

$$\cos\gamma = \frac{55^2+276^2-330^2}{2\cdot 55\cdot 276} = -0.9782$$

$$\gamma \approx 168.02^\circ$$

Pilot needs to turn through $180^\circ - 168.02^\circ = 11.98^\circ$

b) new avg. speed is $\frac{276}{1.25} = 220.80$ mph

75 minutes (1.25 hr) to travel 276 miles