Spring '06/MAT 145/Worksheet 5 **Name:** Solution *Show all your work.*

1. (6pts) Suppose that $\pi < \alpha < \frac{3\pi}{2}$ and $-\frac{\pi}{2} < \beta < 0$ are angles so that $\cos\alpha = -\frac{2}{3}$ and $\cos\beta = \frac{1}{3}$. Find the exact value of $\sin(\alpha+\beta)$.

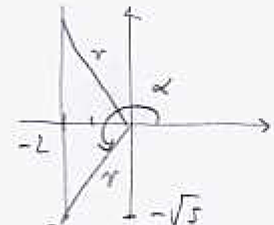
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= -\frac{\sqrt{5}}{3}\cdot\frac{1}{3} + \left(-\frac{2}{3}\right)\left(-\frac{\sqrt{8}}{3}\right)$$
$$= \frac{-\sqrt{5}+2\sqrt{8}}{9} = \frac{4\sqrt{2}-\sqrt{5}}{9}$$
$$\sqrt{8} = \sqrt{4\cdot 2} = 2\sqrt{2}$$

$\cos\alpha = -\frac{2}{3} = \frac{x}{r}$

$(-2)^2 + y^2 = 3^2$

$y^2 = 5$

$y = \pm\sqrt{5}$ $\quad y = -\sqrt{5}$

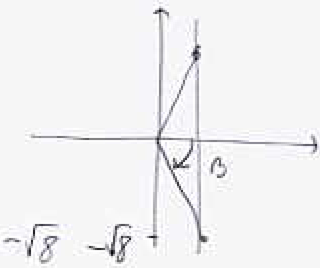


$\cos\beta = \frac{1}{3} = \frac{x}{r}$

$1^2 + y^2 = 3^2$

$y^2 = 8$

$y = \pm\sqrt{8}$ $\quad y = -\sqrt{8}$



2. (4pts) Find the exact value of $\tan 22.5^\circ$ (do not use the calculator).

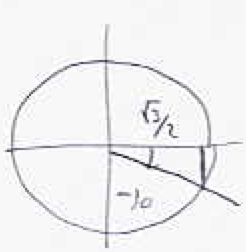
$$\tan^2 22.5 = \frac{1-\cos 45^\circ}{1+\cos 45^\circ} = \frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}\cdot\frac{2}{2} = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

$$\tan 22.5 = \pm\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$$

+ b/c $\tan 22.5^\circ > 0$

3. (3pts) Find the exact value of the expression (do not use the calculator):

$\cos 22^\circ \cos 52^\circ + \sin 22^\circ \sin 52^\circ = \cos(22^\circ - 52^\circ) = \cos(-30^\circ) = \frac{\sqrt{3}}{2}$



Show the following identities.

4. (4pts) $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$$\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \frac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta} = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \tan\alpha + \tan\beta$$

5. (5pts) $\sin^2\theta\cos^2\theta = \dfrac{1}{8}(1-\cos(4\theta))$

half-angle formula

$$\frac{1}{8}\left(1-\cos(4\theta)\right) = \frac{1}{4}\,\frac{1-\cos(4\theta)}{2} = \frac{1}{4}\sin^2\left(\frac{4\theta}{2}\right) = \frac{1}{4}\sin^2(2\theta)$$

$$= \frac{1}{4}\left(2\sin\theta\cos\theta\right)^2 = \frac{1}{\cancel{4}}\cancel{4}\sin^2\theta\cos^2\theta = \sin^2\theta\cos^2\theta$$

double angle formula

6. (8pts) Find the exact value (do not use the calculator) of $\cos\dfrac{5\pi}{12}$ in two ways. (Your answers may differ how they look, but they should be the same number, which you may check using the calculator.)

a) Use the addition formula.

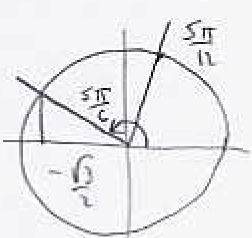
b) Use a half-angle formula.

a) $$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4} \approx 0.2588$$

b) $$\cos^2\left(\frac{5\pi}{12}\right) = \cos^2\left(\frac{\frac{5\pi}{6}}{2}\right) = \frac{1+\cos\frac{5\pi}{6}}{2} = \frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}\cdot\frac{2}{2} = \frac{2-\sqrt{3}}{4}$$

are equal



$$\cos\left(\frac{5\pi}{12}\right) = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2} \approx 0.2588$$

+ since $\frac{5\pi}{12}$ is in 1st quadrant.