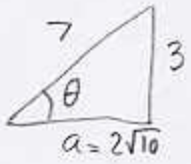
Spring '06/MAT 145/Worksheet 2 **Name:** Solution *Show all your work.*

1. (5pts) If θ is an acute angle, find the values of $\cos\theta$, $\csc\theta$ and $\cot\theta$ given that $\sin\theta = \frac{3}{7}$.

$\sin\theta = \frac{opp}{hyp} = \frac{3}{7}$



$a^2 + 3^2 = 7^2$ $a = \sqrt{40} = \sqrt{4\cdot 10}$

$a^2 + 9 = 49$ $= 2\sqrt{10}$

$a^2 = 40$

$\cos\theta = \frac{2\sqrt{10}}{7}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{3}$

$\cot\theta = \frac{adj}{opp} = \frac{2\sqrt{10}}{3}$

2. (4pts) Use fundamental identities and complementary angles to simplify:

$\sin 31^\circ \cos 59^\circ + \cos 31^\circ \sin 59^\circ =$ $\sin 31^\circ \sin 31^\circ + \cos 31^\circ \cos 31^\circ = \sin^2 31^\circ + \cos^2 31^\circ$

$= 1$

$1 + \tan^2 15^\circ - \csc^2 75^\circ =$ $\underbrace{1 + \tan^2 15^\circ}_{= \sec^2 15^\circ} - \sec^2 15^\circ = \sec^2 15^\circ - \sec^2 15^\circ = 0$

3. (5pts) Find the exact value of each expression. Do not use the calculator.

$\sin^2 60^\circ + \tan^2 30^\circ =$ $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2 = \frac{3}{4} + \left(\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\right)^2 = \frac{3}{4} + \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{3}{4} + \frac{1}{3} = \frac{9+4}{12} = \frac{13}{12}$

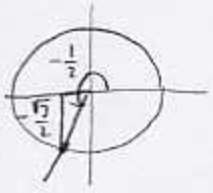
$\left(\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\right)\cot\frac{\pi}{3} = \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)\cdot\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}-1}{2}\cdot\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{\sqrt{2}-1}{2\sqrt{3}}$

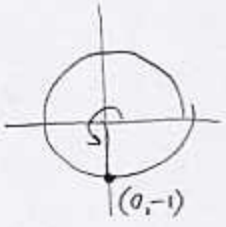
4. (3pts) Use your calculator to evaluate (round to 4 decimals):

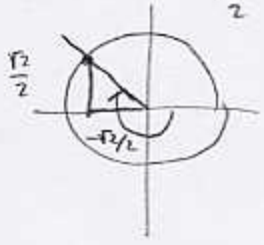
$\cot 223^\circ = \dfrac{1}{\tan 223^\circ} \approx 1.0724$ $\quad \csc\dfrac{3\pi}{7} = \dfrac{1}{\sin\frac{3\pi}{7}} \approx 1.0257$

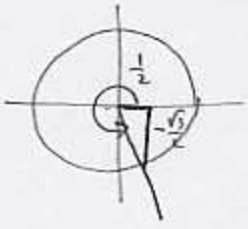
5. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$\cos 240^\circ = -\dfrac{1}{2}$ $\quad \tan\dfrac{3\pi}{2} = \dfrac{-1}{0}$ not defined $\quad \csc(-225^\circ) = \dfrac{1}{\sin(-225^\circ)} = \dfrac{1}{\frac{\sqrt{2}}{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$ $\quad \cot\dfrac{5\pi}{3} = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \dfrac{1}{2}\cdot\left(-\dfrac{2}{\sqrt{3}}\right) = -\dfrac{1}{\sqrt{3}}$





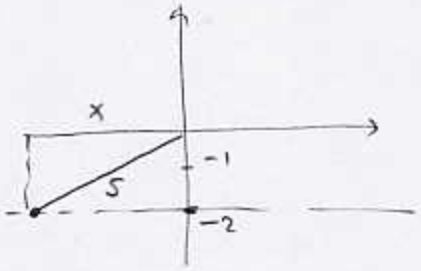




6. (5pts) If $\sin\theta = -\frac{2}{5}$ and θ is in the third quadrant, find $\cos\theta$, $\cot\theta$, $\sec\theta$. Draw a picture.

$r = 5$

$y = -2$



$\cos\theta = \dfrac{-\sqrt{21}}{5}$

$\cot\theta = \dfrac{-\sqrt{21}}{-2} = \dfrac{\sqrt{21}}{2}$

$\sec\theta = \dfrac{1}{\cos\theta} = -\dfrac{5}{\sqrt{21}}$

$x^2 + (-2)^2 = 5^2$

$x^2 + 4 = 25$

$x^2 = 21$

$x = \pm\sqrt{21}$

$x = -\sqrt{21}$

(since in 3rd quad.)