1. (2pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$70° =$

$\dfrac{7\pi}{6}$ radians $=$

2. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$\cos 150° =$ $\tan \dfrac{4\pi}{3} =$ $\sec(-270°) =$ $\sin \dfrac{26\pi}{3} =$

3. (2pts) Use your calculator to evaluate (round to 4 decimals):

$\sec 115° =$ $\tan \dfrac{4\pi}{9} =$

4. (5pts) In a right triangle, the leg opposite angle θ has length 3 and the hypothenuse has length 8. Compute $\cos\theta$, $\csc\theta$ and $\tan\theta$.

5. (5pts) Use fundamental identities and complementary angles to simplify:

$$\frac{\sin 40°}{\sin 50°} - \tan 40° =$$
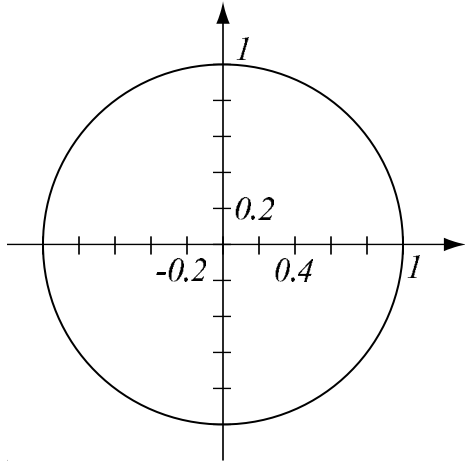
$$\sin 65° \csc 65° + \cos 41° \csc 49° =$$

6. (4pts) Use the odd-even and periodicity properties to figure out:
a) If $\sin\theta = 0.7$, what is $\sin(-\theta)$?
b) If $\cos\theta = -0.35$, what is $\cos(-\theta)$?
c) If $\sin\theta = 0.15$, what is $\sin\theta + 2\sin(\theta + 4\pi) - 3\sin(\theta - 6\pi)$?

7. (4pts) Use the picture below to estimate $\sin \frac{5\pi}{8}$ and $\cos \frac{5\pi}{8}$. Compare your answer with results you get with a calculator.

estimate   calculator

$$\cos \left(\frac{5\pi}{8} \right) =$$

$$\sin \left(\frac{5\pi}{8} \right) =$$

8. (5pts) If $\cos \theta = \frac{1}{3}$ and θ is in the fourth quadrant, find $\sin \theta$, $\cot \theta$, $\sec \theta$. Draw a picture.

9. (5pts) A Ferris wheel of diameter 70ft has rotated $35°$ between two stops. What is the distance (length of arc) that a point on the rim of the Ferris wheel has traveled?

10. (5pts) The Moon revolves around Earth on an approximately circular orbit with radius 384,400km. What is the Moon's linear speed (in km/hr) if it completes one full revolution in 27.32 days?

11. (5pts) A ship is just offshore of New York City. A sighting is taken of the Statue of Liberty, which about 305 feet tall. If the angle of elevation to the top of the statue is 20°, how far is the ship from the base of the statue?

Bonus. (5pts) Find the area of a right triangle, if its hypothenuse is 6cm and one of its angles is 37°.

1. (6pts) Draw two periods of the graph of $y = 2\cos(3x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).
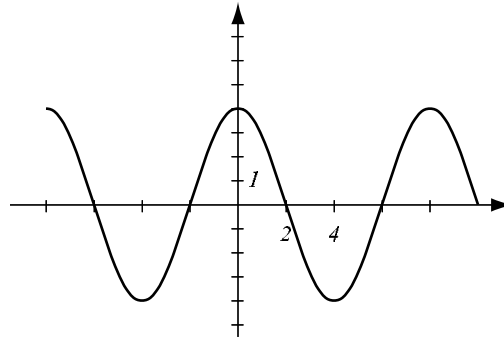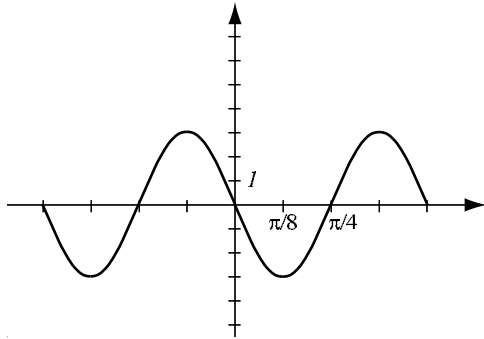
2. (5pts) Draw two periods of the graph of $y = \tan(2x)$. What is the period? Indicate where the special points are (x-intercepts, asymptotes).

3. (8pts) For each of the following two graphs, do the following:
a) Find the amplitude.
b) Find the period.
c) Use this information to help you find the equation for each graph.

4. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$$\arcsin\left(-\frac{1}{2}\right) = \qquad \arcsin\left(-\frac{\sqrt{2}}{2}\right) = \qquad \arctan(-1) = \qquad \arccos(1.25) =$$

5. (5pts) Use the picture below to estimate (in degrees) the values of inverse trigonometric functions. Compare your answer with results you get with a calculator.
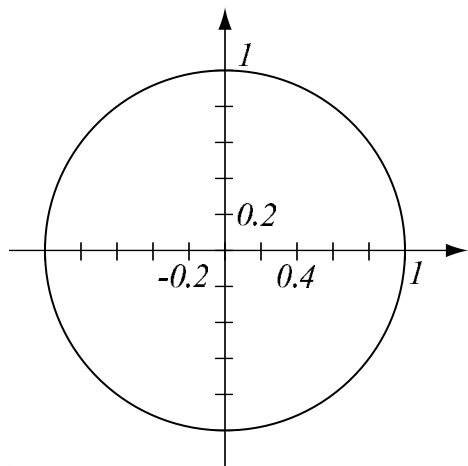
	estimate	calculator
$\arcsin(-0.6) =$		
$\arctan 0.8 =$		

6. (4pts) Simplify the following expressions without using the calculator. For one of them, you will need a picture.

$\cos(\arccos(-0.78)) =$

$\arcsin\left(\sin\dfrac{9\pi}{7}\right) =$

7. (4pts) Evaluate the following expression without using the calculator. Draw the appropriate picture.

$\cos\left(\arcsin\dfrac{4}{5}\right) =$

Use basic trigonometric identities to establish the following identities:

8. (5pts) $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

9. (5pts) $1 - \dfrac{\sin^2\theta}{1 + \cos\theta} = \cos\theta$

Bonus. (5pts) Sir Edmund Hillary is driving on a straight road down a mountain. Between two checks of his GPS instrument, he traveled a distance of 5 miles while his altitude dropped 2000ft. What is the angle between the road and the horizontal?

1. (5pts)   Use an addition formula to find the exact value of $\cos 165°$ (do not use the calculator).

2. (3pts) Find the exact value of the expression (do not use the calculator):

$$\frac{\tan 57° + \tan 12°}{1 - \tan 57° \tan 12°} =$$

3. (5pts)   Use a half-angle formula to find the exact value of $\cos \dfrac{7\pi}{12}$ (do not use the calculator).

4. (9pts) Suppose that $-\dfrac{\pi}{2} < \alpha < 0$ and $\dfrac{\pi}{2} < \beta < \pi$ are angles so that $\tan \alpha = -\dfrac{5}{3}$ and $\sin \beta = \dfrac{2}{7}$. Use addition and half-angle formulas to find:

a) $\sin(\alpha - \beta)$

b) $\sin \dfrac{\alpha}{2}$

5. (4pts) Solve the equation (give a general formula for all the solutions).

$2 \cos \theta + \sqrt{3} = 0$

6. (5pts) Use your calculator to solve the equation on the interval $0 \le \theta < 2\pi$. Round answers to two decimal places (answers in radians).

$\sin \theta = 0.7$

7. (6pts) Solve the equation on the interval $-\pi \le \theta \le \pi$:

$\sin \theta + \cos \theta = 0$

8. (8pts) Solve the equation (give a general formula for all the solutions).

$\sin \theta + 1 = 2 \cos^2 \theta$

9. (5pts) Establish the identity: $\dfrac{1 - \tan^2 \dfrac{\theta}{2}}{1 + \tan^2 \dfrac{\theta}{2}} = \cos\theta$

Bonus. (5pts) Establish the identity $\tan(\pi - \theta) = -\tan\theta$ in two ways:
a) by using an addition formula
b) by drawing a picture and explaining.

1. (5pts) Solve a right triangle if $a = 4$ and $b = 7$.

2. (7pts) Solve the triangle: $a = 5$, $\alpha = 39°$, $\gamma = 105°$.

3. (10pts) Solve the triangle: $c = 8$, $b = 11$, $\beta = 47°$.

4. (8pts) A blimp, suspended in the air at height of 500ft, lies directly over the line between a museum and a park. If the angle of depression to the museum is $35°$ and the angle of depression to the park is $19°$, how far is the museum from the park?
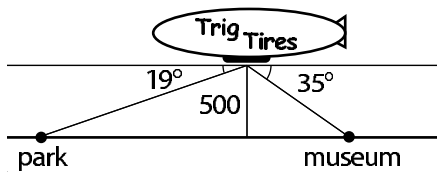
5. (8pts) An office building has a triangular base with sides 112ft, 200ft and 160ft.
a) The boss in a company with offices in the building wishes to have a corner office in the corner of the building with the sharpest angle. What is this angle?
b) What is the square footage of a floor of this office building?

6. (8pts) You take a sighting of the top of a building from a certain point and find that the angle of elevation is 33°. Then you move 200ft towards the building and take another sighting, finding the angle of elevation to be 51° now.
a) How tall is the building?
b) How far were you from the building on the second sighting?

7. (4pts) Give an example of data a, b, β where the SSA triangle does not have a solution. Draw a picture and explain.

Bonus (5pts) How fast (in mph) are people in Barrow, Alaska, moving due to Earth's rotation? Barrow is at $70°$ north latitude and radius of Earth is 3960mi (recall $v = r\omega$, $\omega = \theta/t$).