Spring '06/MAT 145/Final Exam **Name:** Solution *Show all your work.*

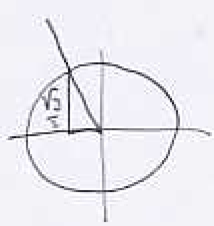
1. (2pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

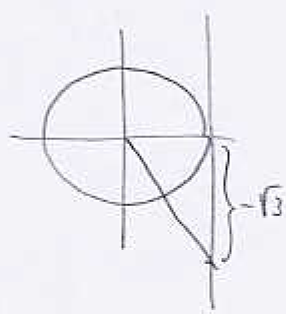
$55° = 55 \cdot \frac{\pi}{180}$ radians $= \frac{11\pi}{36} = 0.96$ radians

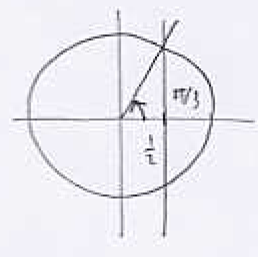
$\frac{7\pi}{6}$radians $= \frac{7\cancel{\pi}^{1}}{\cancel{6}_{1}} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}_{1}} = 210°$

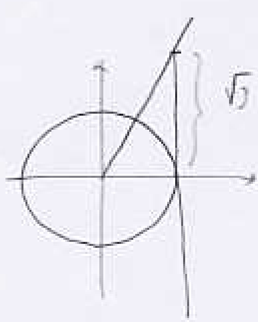
2. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle to help you.

$\sin 120° = \frac{\sqrt{3}}{2}$ $\qquad \cot\frac{5\pi}{3} = \frac{1}{\tan\frac{5\pi}{3}} = -\frac{1}{\sqrt{3}}$ $\qquad \arccos\frac{1}{2} = \frac{\pi}{3}$ $\qquad \arctan\sqrt{3} = \frac{\pi}{3}$

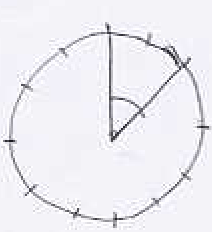








3. (5pts) What is the distance (length of arc) that the tip of a 7-inch minute hand travels in 10 minutes?

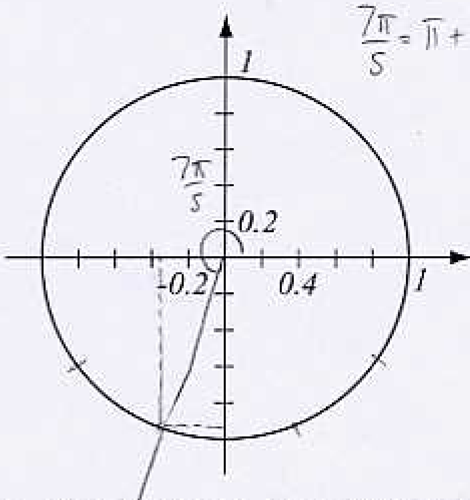


In 10 minutes the minute hand sweeps $\frac{2}{12}$ of 2π radians

$s = r\theta = 7 \cdot \underset{\substack{\uparrow \\ \frac{2}{12} = \frac{1}{6}}}{\frac{1}{6}} \cdot 2\pi = \frac{7\pi}{3} = 7.33$ in

4. (4pts) Use the picture below to estimate $\sin\frac{7\pi}{5}$ and $\cos\frac{7\pi}{5}$. Compare your answer with results you get with a calculator.

$\frac{7\pi}{5} = \pi + \frac{2\pi}{5}$



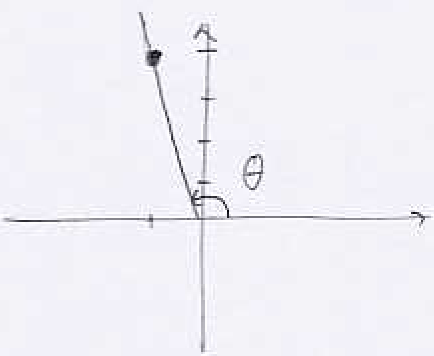
	estimate	calculator
$\cos\frac{7\pi}{5} =$	-0.35	-0.31
$\sin\frac{7\pi}{5} =$	-0.9	-0.95

5. (5pts) If $\tan\theta = -4$ and θ is in the second quadrant, find $\sin\theta$, $\cot\theta$, $\sec\theta$. Draw a picture.

$\tan\theta = \frac{y}{x} = \frac{4}{-1}$

$r = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$

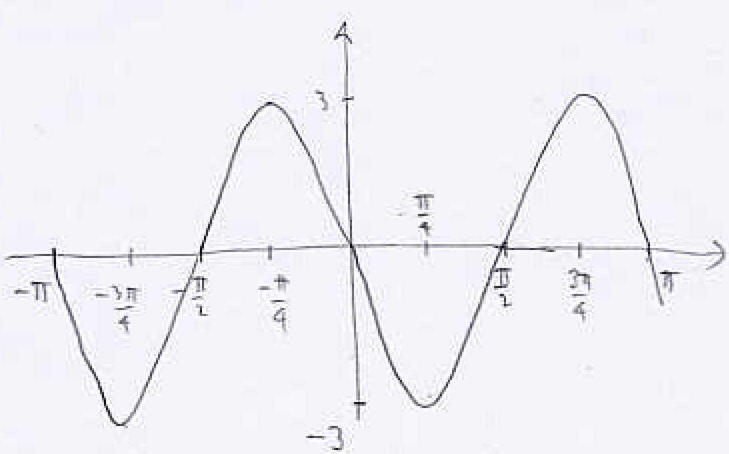
$\sin\theta = \frac{y}{r} = \frac{4}{\sqrt{17}}$



$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{4}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{\sqrt{17}}{-1} = -\sqrt{17}$

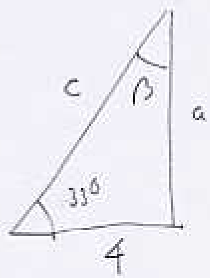
6. (5pts) Draw two periods of the graph of $y = -3\sin(2x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).



Amplitude = 3

Period = $\frac{2\pi}{2} = \pi$

7. (5pts) Solve a right triangle if $b = 4$ and $\alpha = 33°$.



$\frac{4}{c} = \cos 33°$

$\frac{4}{\cos 33°} = c$

$c = 4.77$

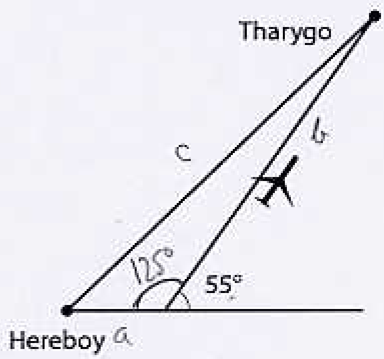
$\beta = 90° - 33° = 57°$

$\frac{a}{4} = \tan 33°$

$a = 4 \tan 33°$

$= 2.60$

8. (5pts) An airplane intended to fly from Hereboy to Tharygo, but made an error and started out in the wrong direction. After 15 minutes, the pilot noticed the error, turned the plane through 55° and continued for another 90 minutes to Tharygo. If the airplane flew at 200mph the whole time, how far is Hereboy from Tharygo?



$a = 200\text{ mph} \cdot \frac{1}{4}\text{ hr} = 50\text{ mi}$

$b = 200\text{ mph} \cdot 1.5\text{ hr} = 300\text{ mi}$

$c^2 = 50^2 + 300^2 - 2 \cdot 50 \cdot 300 \cos 125°$

$= 109707.29$

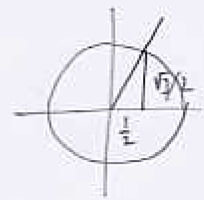
$c = 331.22$ mi ← distance from Hereboy to Tharygo

9. (5pts) Use an addition formula to find the exact value (do not use the calculator):

$$\cos\frac{7\pi}{12} = \cos\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$

$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

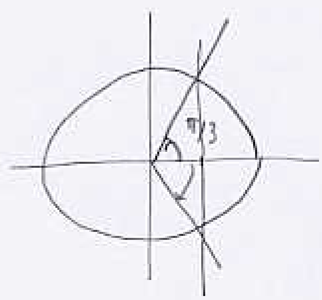


10. (4pts) Solve the equation (give a general formula for all the solutions).

$2\cos\theta - 1 = 0$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$



$$\theta = \frac{\pi}{3} + k\cdot 2\pi$$

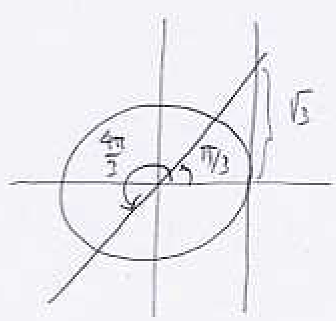
$$\text{or } -\frac{\pi}{3} + k\cdot 2\pi$$

11. (5pts) Solve the equation on the interval $0 \leq \theta \leq 2\pi$:

$\sin\theta - \sqrt{3}\cos\theta = 0$

$$\sin\theta = \sqrt{3}\cos\theta \quad / \div \cos\theta$$

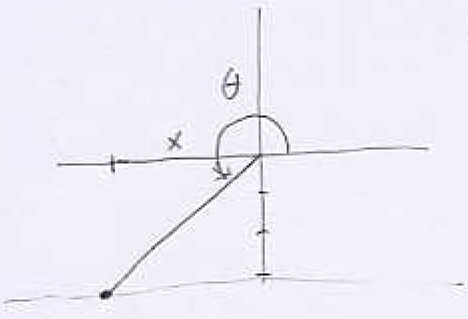
$$\frac{\sin\theta}{\cos\theta} = \sqrt{3}$$



$$\tan\theta = \sqrt{3}$$

$$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$$

12. (5pts) Suppose $\sin\alpha = -\frac{3}{5}$ and α is in the third quadrant. Use a half-angle formula to find the exact value of $\cos\frac{\alpha}{2}$ (do not use the calculator).



$\sin\alpha = \frac{y}{r} = \frac{-3}{5}$

$x^2 + (-3)^2 = 5^2$

$x^2 = 16$

$x = \pm 4$

$x = -4$ by position

$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} = \frac{1 + -\frac{4}{5}}{2} \cdot \frac{5}{5}$

$= \frac{5-4}{10} = \frac{1}{10}$

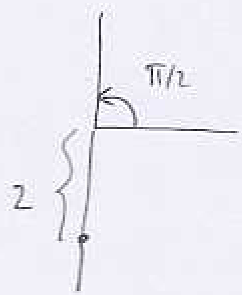
$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1}{10}}$, $\cos\frac{\alpha}{2} = -\sqrt{\frac{1}{10}}$

b/c $\frac{\pi}{2} \le \frac{\alpha}{2} < \pi$

13. (2pts) Sketch points in a polar coordinate system whose polar coordinates (r, θ) are:

$(3, \frac{3\pi}{4})$

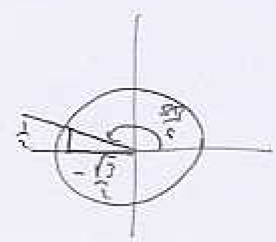
$(-2, \frac{\pi}{2})$.



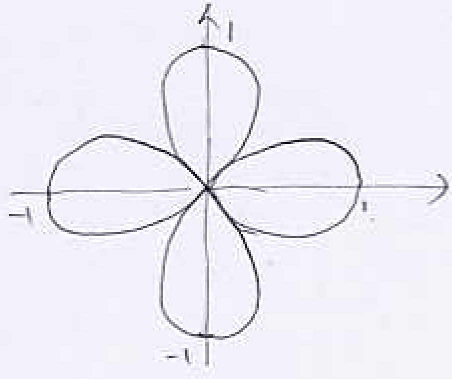
14. (2pts) Find the exact rectangular coordinates of the point whose polar coordinates are $(3, \frac{5\pi}{6})$. Do not use the calculator.

$x = r\cos\theta = 3\cos\frac{5\pi}{6} = 3\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$

$y = r\sin\theta = 3\sin\frac{5\pi}{6} = 3\cdot\frac{1}{2} = \frac{3}{2}$



15. (3pts) Use the graphing feature on your calculator to sketch the polar curve $r = \cos(2\theta)$.



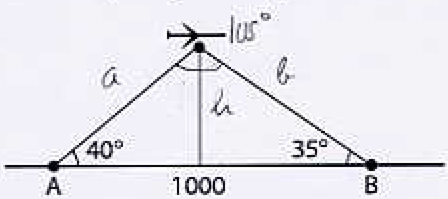
Rose with 4 petals

16. (5pts) Use trigonometric identities to establish the following identity:

$4\cos^2\theta - 4\cos^4\theta = \sin^2(2\theta)$

$$4\cos^2\theta - 4\cos^4\theta = 4\cos^2\theta(1-\cos^2\theta)$$
$$= 4\cos^2\theta\sin^2\theta$$
$$= (2\sin\theta\cos\theta)^2$$
$$= (\sin(2\theta))^2$$
$$= \sin^2(2\theta)$$

Bonus. (7pts) Observers A and B spot an airplane and measure angles of elevation of 40° and 35°, respectively. If the observers are 1000ft apart, how high is the airplane?



$$y = 180° - (40° + 35°)$$
$$= 105°$$

$$\frac{\sin 35°}{a} = \frac{\sin 105°}{1000}$$

$$1000\sin 35° = a\sin 105°$$

$$a = \frac{1000\sin 35°}{\sin 105°} = 593.81 \text{ ft}$$

$$\frac{h}{a} = \sin 40°$$

$$h = a\sin 40°$$
$$= 593.81\cdot\sin 40°$$
$$= 381.69 \text{ ft}$$