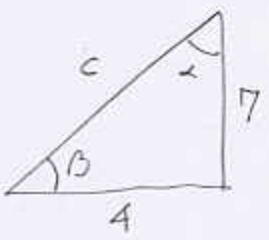
Spring '06/MAT 145/Exam 4 **Name:** Solution *Show all your work.*

1. (5pts) Solve a right triangle if $a = 4$ and $b = 7$.



$\tan\alpha = \frac{4}{7}$

$\alpha = \arctan\alpha = 29.74°$

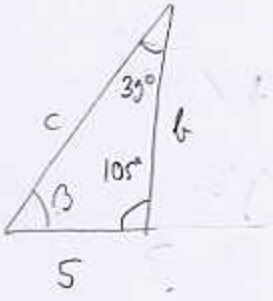
$\beta = 90 - \alpha = 60.26°$

$c^2 = 4^2 + 7^2$

$c^2 = 65$

$c = \sqrt{65} = 8.06$

2. (7pts) Solve the triangle: $a = 5$, $\alpha = 39°$, $\gamma = 105°$.



$\beta = 180° - (39° + 105°)$

$= 36°$

$\frac{\sin 36°}{b} = \frac{\sin 39°}{5}$

$5\sin 36° = b\sin 39°$

$b = \frac{5\sin 36°}{\sin 39°} = 4.67$

$\frac{\sin 105°}{c} = \frac{\sin 39°}{5}$

$5\sin 105° = c\sin 39°$

$c = \frac{5\sin 105°}{\sin 39°} = 7.67$

3. (10pts) Solve the triangle: $c = 8$, $b = 11$, $\beta = 47°$.

$\frac{\sin\gamma}{8} = \frac{\sin 47°}{11}$

$\sin\gamma = \frac{8\sin 47°}{11} = 0.5318..$

$\gamma_1 = 32.13°$ or $\gamma_2 = 147.87°$

$\alpha = 100.87°$ then $\alpha_2 = 180° - (47° + 147.87)°$

$= -14.86°$

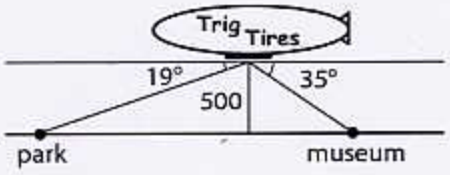
not possible

There is only one solution.

$\frac{\sin 100.87°}{a} = \frac{\sin 47°}{11}$

$a = \frac{11\sin 100.87°}{\sin 47°} = 14.77$

4. (8pts) A blimp, suspended in the air at height of 500ft, lies directly over the line between a museum and a park. If the angle of depression to the museum is 35° and the angle of depression to the park is 19°, how far is the museum from the park?



Need a+b.

$\tan 19° = \frac{500}{a}$ $\tan 35° = \frac{500}{b}$

$a\tan 19° = 500$ $b\tan 35° = 500$

$a = \frac{500}{\tan 19°}$ $b = \frac{500}{\tan 35°}$

$a + b = \frac{500}{\tan 19°} + \frac{500}{\tan 35°} = 2166.18ft$

$= 1452.11 + 714.07$

5. (8pts) An office building has a triangular base with sides 112ft, 200ft and 160ft.
a) The boss in a company with offices in the building wishes to have a corner office in the corner of the building with the sharpest angle. What is this angle?
b) What is the square footage of a floor of this office building?

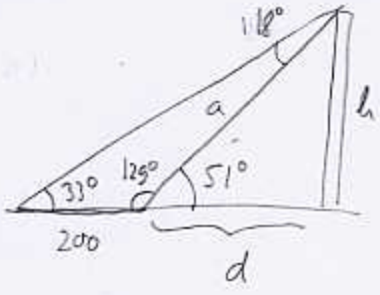
a) smallest angle is opposite smallest side

$$\cos\alpha = \frac{200^2 + 160^2 - 112^2}{2 \cdot 160 \cdot 200} = \frac{53056}{64000} = 0.829$$

$$\alpha = \arccos 0.829 = 34^\circ$$

b) $A = \frac{1}{2}bc\sin\alpha = \frac{1}{2} \cdot 200 \cdot 160 \cdot \sin 34^\circ = 8947.09$ sq.ft

6. (8pts) You take a sighting of the top of a building from a certain point and find that the angle of elevation is 33°. Then you move 200ft towards the building and take another sighting, finding the angle of elevation to be 51° now.
a) How tall is the building?
b) How far were you from the building on the second sighting?



a) Need a first.

$$\frac{\sin 33^\circ}{a} = \frac{\sin 18^\circ}{200}$$

$$200\sin 33^\circ = a\sin 18^\circ$$

$$a = \frac{200\sin 33^\circ}{\sin 18^\circ} = 352.50 \text{ ft}$$

$$\frac{h}{a} = \sin 51^\circ$$

$$h = a\sin 51^\circ$$

$$= 352.5 \cdot \sin 51^\circ$$

$$= 273.94 \text{ ft}$$

↑ height of building

b) $\frac{d}{a} = \cos 51^\circ$

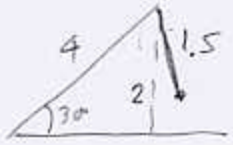
$$d = a\cos 51^\circ = 352.5 \cdot \cos 51^\circ = 221.83 \text{ ft}$$

7. (4pts) Give an example of data a, b, β where the SSA triangle does not have a solution. Draw a picture and explain.

If b is short (shorter than $a \sin\beta$), we won't be able to close the triangle

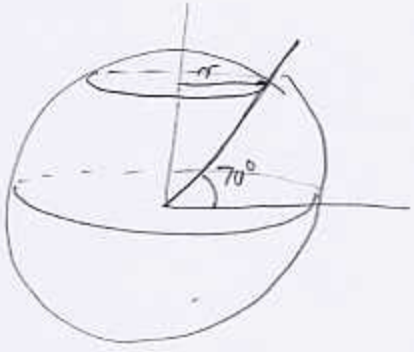
Example: $a = 4$, $\beta = 30°$

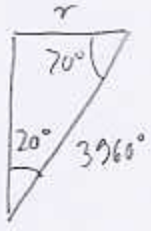
$a \sin\beta = 4 \sin 30° = 4 \cdot \frac{1}{2} = 2$



Take $b = 1.5$

Bonus (5pts) How fast (in mph) are people in Barrow, Alaska, moving due to Earth's rotation? Barrow is at 70° north latitude and radius of Earth is 3960mi (recall $v = r\omega$, $\omega = \theta/t$).





$\frac{r}{3960} = \cos 70°$

$r = 3960 \cos 70°$

$= 1354.40$ mi

Need r.

$v = r\omega = 1354.40 \cdot \frac{2\pi}{24 \text{ hrs}} = \frac{8509.94}{24} = 354$ mph