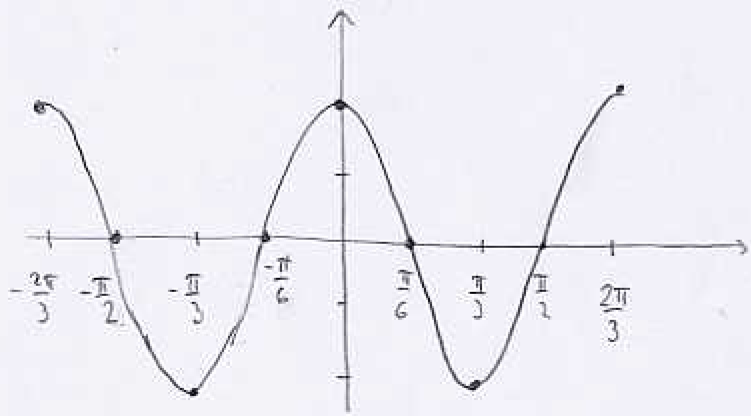
Spring '06/MAT 145/Exam 2 **Name:** Solution *Show all your work.*

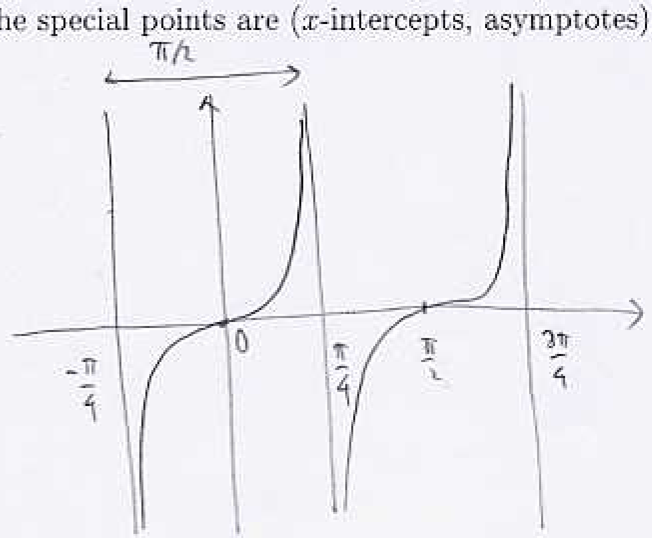
1. (6pts) Draw two periods of the graph of $y = 2\cos(3x)$. What is the amplitude? The period? Indicate where the special points are (x-intercepts, peaks, valleys).

$A = 2$

$P = \frac{2\pi}{3}$

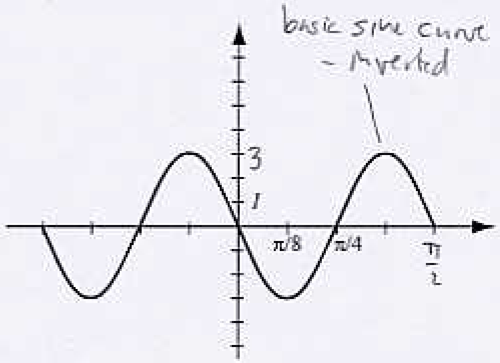


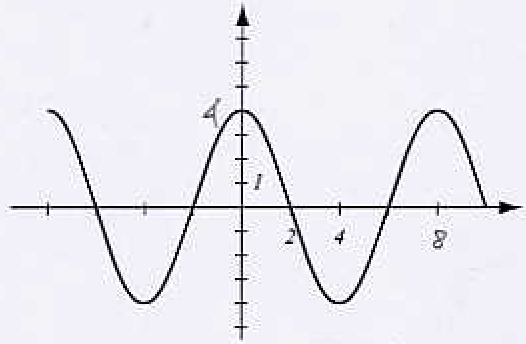
2. (5pts) Draw two periods of the graph of $y = \tan(2x)$. What is the period? Indicate where the special points are (x-intercepts, asymptotes).



$P = \frac{\pi}{2}$

3. (8pts) For each of the following two graphs, do the following:
a) Find the amplitude.
b) Find the period.
c) Use this information to help you find the equation for each graph.





$A = 3$

$P = \frac{\pi}{2}$

$\frac{2\pi}{\omega} = \frac{\pi}{2}$

$4\pi = \pi\omega$

$\omega = \frac{4\pi}{\pi} = 4$

$A = 4$

$P = 8$

$\frac{2\pi}{\omega} = 8$

$2\pi = 8\omega$

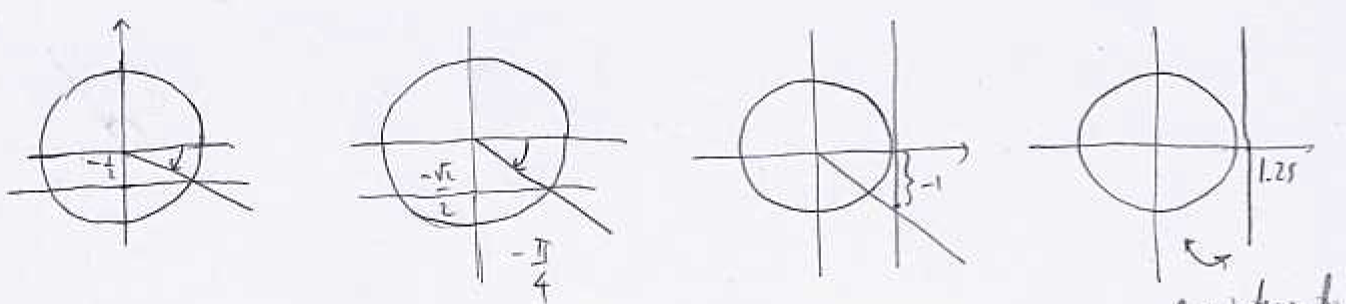
$\omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$y = -3\sin(4x)$

$y = 4\cos\left(\frac{\pi}{4}x\right)$

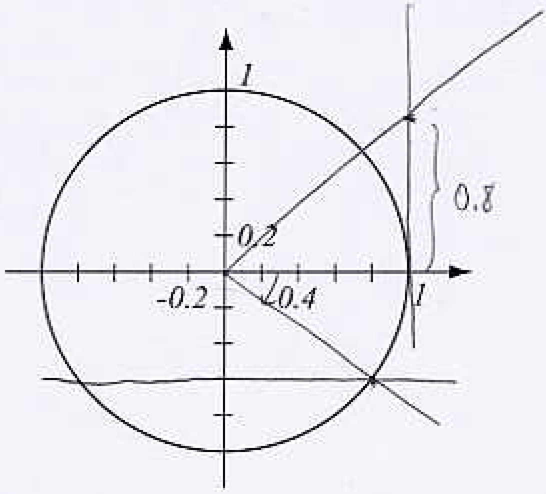
4. (8pts) Without using the calculator, find the exact values (in radians) of the following expressions. Draw the unit circle to help you.

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ $\quad$ $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ $\quad$ $\arctan(-1) = -\frac{\pi}{4}$ $\quad$ $\arccos(1.25) =$ not defined



no intersection with unit circle

5. (5pts) Use the picture below to estimate (in degrees) the values of inverse trigonometric functions. Compare your answer with results you get with a calculator.

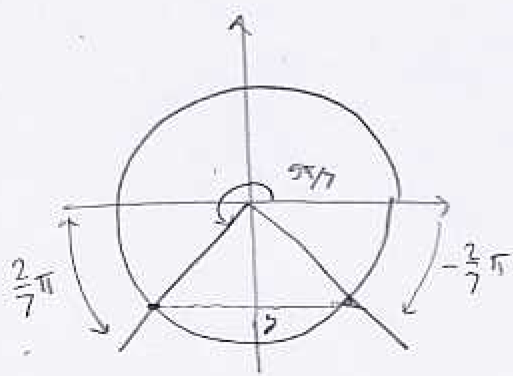


	estimate	calculator
$\arcsin(-0.6) =$	-40°	-36.87°
$\arctan 0.8 =$	40°	38.65°

6. (4pts) Simplify the following expressions without using the calculator. For one of them, you will need a picture.

$\cos(\arccos(-0.78)) = -0.78$

$\arcsin\left(\underbrace{\sin\frac{9\pi}{7}}_{y}\right) = \arcsin y = -\frac{2\pi}{7}$

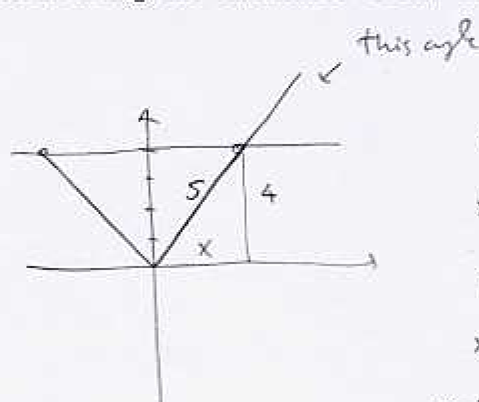


7. (4pts) Evaluate the following expression without using the calculator. Draw the appropriate picture.

$\cos\left(\underbrace{\arcsin\frac{4}{5}}_{\theta}\right) = \cos\theta = \frac{x}{r} = \frac{3}{5}$

$\sin\theta = \frac{4}{5} = \frac{y}{r}$

θ btw $-\frac{\pi}{2}, \frac{\pi}{2}$



$x^2 + 4^2 = 5^2$

$x^2 + 16 = 25$

$x^2 = 9$

$x = \pm 3$

$x = 3$ owing to choice of angle

Use basic trigonometric identities to establish the following identities:

8. (5pts) $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$$\sin\theta(\cot\theta + \tan\theta) = \sin\theta\cdot\cot\theta + \sin\theta\tan\theta$$

$$= \sin\theta\cdot\frac{\cos\theta}{\sin\theta} + \sin\theta\frac{\sin\theta}{\cos\theta}$$

$$= \cos\theta + \frac{\sin^2\theta}{\cos\theta} = \frac{\overbrace{\cos^2\theta + \sin^2\theta}^{1}}{\cos\theta}$$

$$= \frac{1}{\cos\theta} = \sec\theta$$

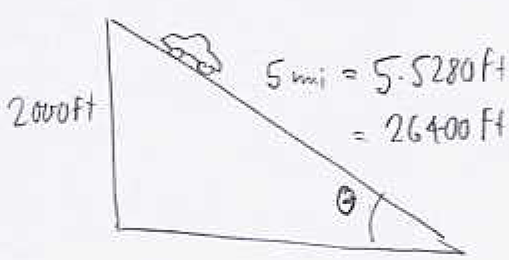
9. (5pts) $1 - \dfrac{\sin^2\theta}{1+\cos\theta} = \cos\theta$

$(1-\sin^2\theta = \cos^2\theta)$

$$1 - \frac{\sin^2\theta}{1+\cos\theta} = \frac{1+\cos\theta - \sin^2\theta}{1+\cos\theta} = \frac{\cos^2\theta + \cos\theta}{1+\cos\theta} = \frac{\cos\theta\cancel{(\cos\theta+1)}}{\cancel{1+\cos\theta}}$$

$$= \cos\theta$$

Bonus. (5pts) Sir Edmund Hillary is driving on a straight road down a mountain. Between two checks of his GPS instrument, he traveled a distance of 5 miles while his altitude dropped 2000ft. What is the angle between the road and the horizontal?



$$\sin\theta = \frac{2000}{26400}$$

$$\sin\theta = 0.0757575\ldots$$

$$\theta = \arcsin 0.0757575\ldots$$

$$\theta = 4.34^\circ$$