

1. (2pts) Convert into the other angle measure (radians or degrees). Show how you computed your number.

$$70^\circ = 70^\circ \cdot \frac{\pi}{180^\circ} = \frac{7\pi}{18} \text{ radians}$$

$$\frac{7\pi}{6} \text{ radians} = \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$$

2. (8pts) Without using the calculator, find the exact values of the following trigonometric expressions. Draw the unit circle and the appropriate angle under the expression.

$$\cos 150^\circ = -\frac{\sqrt{3}}{2}$$

$$\tan \frac{4\pi}{3} = \frac{-\sqrt{3}}{-1}$$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sec(-270^\circ) =$$

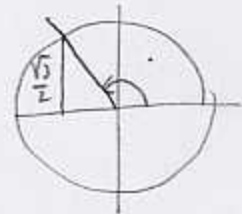
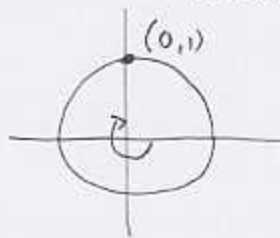
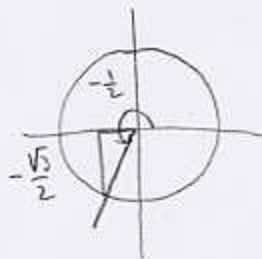
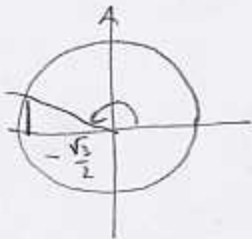
$$= \frac{1}{\cos(-270^\circ)} = \frac{1}{0}$$

undefined

$$\sin \frac{26\pi}{3} = \sin\left(8\pi + \frac{2}{3}\pi\right)$$

$$= \sin\left(\frac{2}{3}\pi\right) = \frac{\sqrt{3}}{2}$$

$$\frac{26}{3} = 8\frac{2}{3}$$



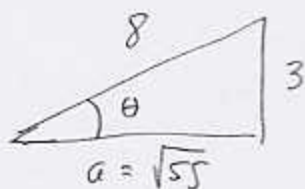
3. (2pts) Use your calculator to evaluate (round to 4 decimals):

$$\sec 115^\circ = \frac{1}{\cos 115^\circ}$$

$$\tan \frac{4\pi}{9} = 5.6713$$

$$= -2.3662$$

4. (5pts) In a right triangle, the leg opposite angle θ has length 3 and the hypotenuse has length 8. Compute $\cos \theta$, $\csc \theta$ and $\tan \theta$.



$$a^2 + 3^2 = 8^2$$

$$a^2 = 64 - 9$$

$$a^2 = 55$$

$$\cos \theta = \frac{\sqrt{55}}{8}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{3}{8}} = \frac{8}{3}$$

$$\tan \theta = \frac{3}{\sqrt{55}}$$

5. (5pts) Use fundamental identities and complementary angles to simplify:

$$\frac{\sin 40^\circ}{\sin 50^\circ} - \tan 40^\circ = \frac{\sin 40^\circ}{\cos 40^\circ} - \tan 40^\circ = \tan 40^\circ - \tan 40^\circ = 0$$

$$\begin{aligned} \sin 65^\circ \csc 65^\circ + \cos 41^\circ \csc 49^\circ &= \cancel{\sin 65^\circ} \cdot \frac{1}{\cancel{\sin 65^\circ}} + \cos 41^\circ \cdot \frac{1}{\sin 49^\circ} \\ &= 1 + \cancel{\cos 41^\circ} \cdot \frac{1}{\cancel{\cos 41^\circ}} = 1 + 1 = 2 \end{aligned}$$

6. (4pts) Use the odd-even and periodicity properties to figure out:

a) If $\sin \theta = 0.7$, what is $\sin(-\theta)$?

b) If $\cos \theta = -0.35$, what is $\cos(-\theta)$?

c) If $\sin \theta = 0.15$, what is $\sin \theta + 2 \sin(\theta + 4\pi) + 3 \sin(\theta - 6\pi)$?

a) $\sin(-\theta) = -\sin \theta = -0.7$

b) $\cos(-\theta) = \cos \theta = -0.35$

c) $\sin \theta + 2 \sin(\theta + 4\pi) + 3 \sin(\theta - 6\pi)$

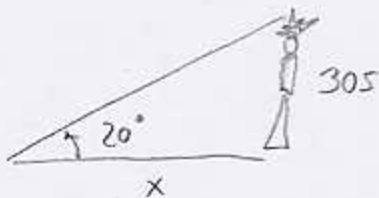
$$= \sin \theta + 2 \sin \theta + 3 \sin \theta = 0$$

10. (5pts) The Moon revolves around Earth on an approximately circular orbit with radius 384,400km. What is the Moon's linear speed (in km/hr) if it completes one full revolution in 27.32 days?

$$v = \omega r = \frac{2\pi}{27.32 \text{ days}} \cdot 384,400 = \frac{2\pi}{27.32 \cdot 24} \cdot 384,400$$

$$= 3683.59 \text{ km/hr}$$

11. (5pts) A ship is just offshore of New York City. A sighting is taken of the Statue of Liberty, which about 305 feet tall. If the angle of elevation to the top of the statue is 20° , how far is the ship from the base of the statue?

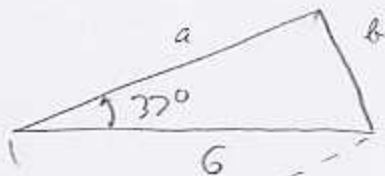


$$\frac{305}{x} = \tan 20^\circ$$

$$305 = x \tan 20^\circ$$

$$x = \frac{305}{\tan 20^\circ} = 837.98 \text{ ft}$$

Bonus. (5pts) Find the area of a right triangle, if its hypotenuse is 6cm and one of its angles is 37° .



$$\frac{a}{6} = \cos 37^\circ \quad \frac{b}{6} = \sin 37^\circ$$

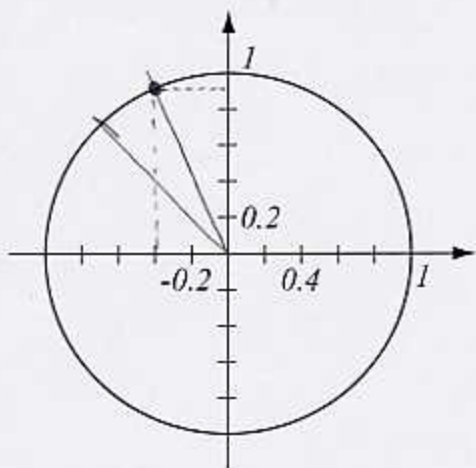
$$a = 6 \cos 37^\circ \quad b = 6 \sin 37^\circ$$

$$A = \frac{ab}{2} = \frac{6 \cos 37^\circ \cdot 6 \sin 37^\circ}{2}$$

$$= 18 \sin 37^\circ \cos 37^\circ$$

$$= 8.65 \text{ cm}^2$$

7. (4pts) Use the picture below to estimate $\sin \frac{5\pi}{8}$ and $\cos \frac{5\pi}{8}$. Compare your answer with results you get with a calculator.



	estimate	calculator
$\cos \left(\frac{5\pi}{8} \right) =$	-0.4	-0.38
$\sin \left(\frac{5\pi}{8} \right) =$	0.9	0.92

8. (5pts) If $\cos \theta = \frac{1}{3}$ and θ is in the fourth quadrant, find $\sin \theta$, $\cot \theta$, $\sec \theta$. Draw a picture.

$$\cos \theta = \frac{1}{3} = \frac{x}{r} \quad \begin{matrix} x=1 \\ r=3 \end{matrix}$$

$$1^2 + y^2 = 3^2$$

$$y^2 = 8$$

$$y = \pm \sqrt{8}$$

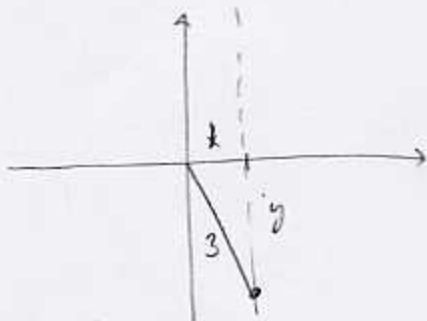
$$y = -\sqrt{8} = -2\sqrt{2}$$

(4th quadrant)

$$\sin \theta = -\frac{\sqrt{8}}{3}$$

$$\cot \theta = -\frac{1}{\sqrt{8}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{1}{3}} = 3$$



9. (5pts) A Ferris wheel of diameter 70ft has rotated 35° between two stops. What is the distance (length of arc) that a point on the rim of the Ferris wheel has traveled?



$$35^\circ = 35 \cdot \frac{\pi}{180} \text{ radians} = \frac{7\pi}{36}$$

$$s = r\theta = 35 \cdot \frac{7\pi}{36} = 21.38 \text{ ft}$$