Fall '06/MAT 220/Worksheet 1 **Name:** *Show all your work.*

1. (2pts) Find the domain of the function $f(x) = \dfrac{x+2}{x-4}$.

2. (5pts) Sketch the following four lines on the same graph:
a) $y = 3x - 2$
b) $2x + 5y = 7$
c) $y = 4$
d) $x = 3$

3. (7pts) A company that manufactures chairs finds that its daily cost is \$32,000 when it manufactures 500 chairs in a day, and its daily cost is \$68,000 if it manufactures 2,000 chairs in a day.
a) Assuming the cost function $C(x)$ is linear, write an equation for the cost function.
b) What is the expected cost for producing 1,400 chairs?
c) Graph the cost function for $0 \leq x \leq 3,000$.

4. (4pts) If $f(x) = x^2 - 3x + 1$ find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

5. (12pts) **Example.** A company produces printers. Analysts at its financial department have found that the price-demand and cost functions are given by

$$p(x) = 500 - 1.5x \qquad C(x) = 85 + 135x \qquad 0 \leq x \leq 200$$

where x is in thousands, p in dollars and C in thousands of dollars.

a) Write the revenue function for this problem and graph it.
b) Find **algebraically** the level of production that maximizes revenue and the maximal revenue.
c) Write the profit function for this problem and graph it.
d) Find **algebraically** the break-even points and check your answer using graphing features on your calculator.

6. (7pts) The polynomial $f(x) = x^4 - 2x^3 - 5x^2 - x - 7$ is given.
a) Use Theorem 2 in 2.1 to find an interval on the x-axis that must contain all the zeros of this polynomial
b) Graph the function on this interval (yes, on paper!).
c) Find all the zeros to three decimal places.
d) Does this polynomial have the maximal number of zeros possible for a fourth-degree polynomial?

7. (5pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

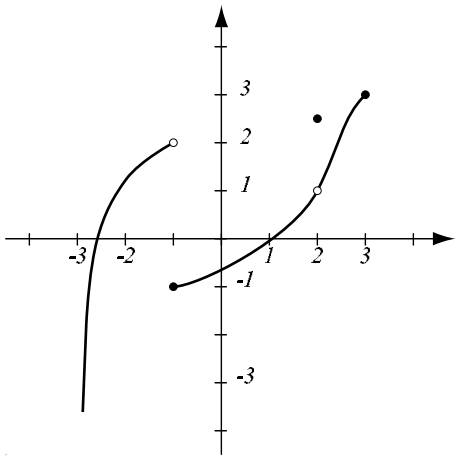
$\lim_{x \to -1^-} f(x) =$

$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to -1} f(x) =$

$\lim_{x \to 2} f(x) =$

$f(2) =$



8. (10pts) The rational function $f(x) = \dfrac{2x^2+6}{x^2-4x+3}$ is given.

a) Find all the x- and y-intercepts for the graph.

b) Find the vertical asymptotes of the graph.

c) Sketch a graph of the function on paper.

d) Find algebraically if the graph has a horizontal asymptote. If so, display it on your graph.

e) $\lim\limits_{x\to 3^+} f(x) =$

f) $\lim\limits_{x\to -\infty} f(x) =$

9. (8pts) Consider the limit $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{x}-\sqrt{5}}$.

a) Use your calculator to make a table of values. Estimate the limit to three decimal places.

b) Find the limit algebraically and compare your answer to a).

10. (9pts) This problem is about finding the tangent line to the curve $f(x) = 4 - x^2$ at the point $P = (1, 3)$.
a) Draw the graph of the function on the interval $[-3, 3]$.
b) Draw three secant lines passing through P and a point Q on the graph, where Q is $(2, f(2))$, $(1.5, f(1.5))$, $(1.25, f(1.25))$ respectively.
c) Find slopes of the lines PQ for each of the three choices and estimate the slope of the tangent line from this information.
d) Use a limit to algebraically find the slope of the tangent line at $(1, 3)$.
e) Write the equation of the desired tangent line.

11. (4pts) Use a limit (i.e. "four-step process") to find $f'(x)$ of the function $f(x) = \sqrt{3 + 2x}$.

12. (10pts) Use differentiation rules to find the derivates:

a) $\dfrac{d}{dx}(4x^3 - 7x^2 + 13x - 6) =$

b) $\dfrac{d}{du}\left(\dfrac{3}{u} - \dfrac{7}{u^6}\right) =$

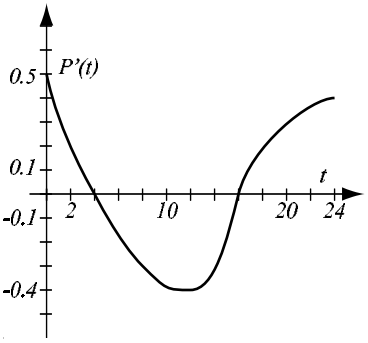
c) $\dfrac{d}{dt}\left(\sqrt{t} + \dfrac{3}{\sqrt{t}}\right) =$

d) $(2\sqrt[4]{x^9} + 5x^{3.4})' =$

13. (7pts) Let $g(x) = 2x^3 - 9x^2 - 60x + 17$.
a) Algebraically find the values of x where the graph of g has a horizontal tangent line.
b) Use your calculator to draw the graph of g (on paper!). Verify your answer from a) on the graph.

14. (10pts) Let $f(x) = (x^2 - 4)^2$.
a) Find the x-intercepts and the y-intercept.
b) Find the intervals of increase and decrease and find the local extrema.
c) Find the intervals where the function is concave up/down and find the inflection points.
d) Sketch a nice graph of the function that takes into account everything you found in a)-c).

15. (10pts) The graph in the figure gives the rate of change $P'(t)$ of the price of electricity $P(t)$ over 24 months. (Units for $P(t)$ are cents/kWh).
a) During which time was the price of electricity increasing? Decreasing?
b) What is $P'(8)$? $P'(18)$? Explain in words the meaning of these numbers.
c) Sketch a possible graph for $P(t)$. Try to make it as accurate as you can. You may assume $P(0) = 9$.



16. (10pts) A company estimates that it will sell $N(x)$ units of a product after spending x thousand on advertising, as given by $N(x) = -0.25x^4 + 13x^3 - 180x^2 + 10000$, $15 \leq x \leq 24$.
a) When is the rate of change of sales increasing? Decreasing?
b) What is the point of diminishing returns and the maximum rate of change of sales?
c) Graph N and N' on the same coordinate system. Where is the inflection point on the graph of N?

17. (11pts) Let $f(x) = \dfrac{2}{x^2+1}$.

a) Find the x-intercepts and the y-intercept.

b) Find the domain and vertical and horizontal asymptotes, if any.

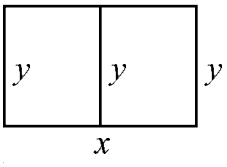
c) Find the intervals of increase and decrease and find the local extrema.

d) Find the intervals where the function is concave up/down and find the inflection points.

e) Sketch a nice graph of the function that takes into account everything you found in a)-d).

18. (9pts) Let $f(x) = 3x^4 + 4x^3 - 72x^2 + 16$. Find the absolute extremes of this function (and where they occur) on the closed intervals below. Then draw on paper a rough graph of the function to verify your answers.
a) $[-2, 4]$
b) $[5, 7]$.

19. (10pts) A farmer will use a fence to enclose a rectangular area and divide it in two parts. If the rectangle is to have area 500m^2, what should be its dimensions in order to minimize the length of the fence used? What is the minimal length of the fence? (When solving this problem, state the domain of your variable and use the second derivative test to ensure that your solution is indeed a minimum.)



20. (4pts) Evaluate without using the calculator:

$\log_3 81 =$ $\log_4 \frac{1}{16} =$ $\log_{34} 1 =$ $\log_a \sqrt[6]{a^5} =$

21. (4pts) Solve the following equations by turning them to exponential form:

$\log_3 x = \frac{1}{2}$ $\log_x 7 = 4$

22. (6pts) Suppose \$2,000 is invested into an account paying 6.21% compounded monthly.
a) How much is in the account in two and a half years?
b) How long will it take until the account is worth \$5,000?

23. (4pts) At what nominal rate compounded continuously must money be invested to double in 3 years?

24. (6pts) The US population is 300 million today and grows according to the model $P = P_0 e^{rt}$ (pg. 313) at the continuous compound rate or growth of 0.85%.
a) Write the function that describes the US population (in millions) t years from now. Then graph the function here.
b) How long until the US population reaches 400 million?

25. (6pts) Radioactive substances decay according to the law $Q = Q_0 e^{rt}$ (pg. 313). A radioactive isotope of carbon, carbon 14, takes 5600 years to decay to half the original amount (that is, the *half-life* of C14 is 5600 years).
a) Find the continuous compound rate r of decay for C14.
b) How long until a sample of C14 decays to one-third of the original amount?

26. (11pts) Find the following derivatives (simplify where possible):

a) $\dfrac{d}{dx}\dfrac{e^x}{x^3-5x^2-17x} =$

b) $\dfrac{d}{dx}\, x^3 e^{x^2+2x-1} =$

c) $\dfrac{d}{dx}\,(\ln(x^2-2))^3 =$

d) $\dfrac{d}{dt}\dfrac{t+1}{\ln t} =$

27. (6pts) The price demand equation for selling x thousand vacuum cleaners is given by $p = 400e^{-0.003x}$, $0 \le x \le 500$. Find the production level and price that maximize revenue. (Use the second-derivative test to check it's a maximum). What is the maximum revenue?

28. (6pts) The demand for bicycles at a sporting-goods store is given by $p = 500 - 20 \ln x$, $0 \leq x \leq 1000$. If the bicycles cost the store \$350 each, how should they be priced to maximize profit? (Use the second-derivative test to check it's a maximum). What is the maximum profit?

29. (7pts) The price-demand equation for T-shirts at a discount store is $p + 3x = 15$, where x thousand T-shirts are sold, $0 \leq x \leq 4$. Use elasticity of demand to solve the following problems.

a) If the current price of \$10.50 is decreased by 8%, by approximately what percentage will demand increase? Will revenue increase or decrease?

b) If the current price of \$6.00 is increased by 5%, by approximately what percentage will demand decrease? Will revenue increase or decrease?