Fall '06/MAT 220/Worksheet 7 **Name:** Solution *Show all your work.*

1. (11pts) Find the following derivatives (simplify where possible):

a) $\dfrac{d}{dx}\dfrac{e^x}{x^3-5x^2-17x} = \dfrac{e^x(x^3-5x^2-17x)-e^x(3x^2-10x-17)}{(x^3-5x^2-17x)^2} = \dfrac{e^x(x^3-8x^2-7x+17)}{(x^3-5x^2-17x)^2}$

b) $\dfrac{d}{dx}x^3e^{x^2+2x-1} = 3x^2e^{x^2+2x-1} + x^3e^{x^2+2x-1}(2x+2) = e^{x^2+2x-1}(3x^2+2x^4+2x^3)$

c) $\dfrac{d}{dx}(\ln(x^2-2))^3 = 3(\ln(x^2-2))^2\cdot\dfrac{1}{x^2-2}\cdot 2x = \dfrac{6x(\ln(x^2-2))^2}{x^2-2}$

d) $\dfrac{d}{dt}\dfrac{t+1}{\ln t} = \dfrac{1\cdot\ln t-(t+1)(\frac{1}{t})}{(\ln t)^2} = \dfrac{\ln t-\frac{t+1}{t}}{(\ln t)^2}\cdot\dfrac{t}{t} = \dfrac{t\ln t-t-1}{t(\ln t)^2}$

2. (6pts) The price demand equation for selling x thousand vacuum cleaners is given by $p = 400e^{-0.003x}$, $0 \le x \le 500$. Find the production level and price that maximize revenue. (Use the second-derivative test to check it's a maximum). What is the maximum revenue?

$R = xp = 400xe^{-0.003x}$

Maximize R on for $0 \le x \le 500$

$R'(x) = 400(1\cdot e^{-0.003x} + xe^{-0.003x}\cdot(-0.003))$

$= \underbrace{400e^{-0.003x}}_{>0}(1-0.003x)$

$R'(x) = 0$ when $1-0.003x = 0$

$1 = 0.003x$

$x = \dfrac{1}{0.003} = 333.333$ vacuum cleaners

$p = 400\cdot e^{-0.003\cdot\frac{1}{0.003}}$

$= 400e^{-1} = \$147.15$

Maximum Revenue $= 49050.59$ thousand

$= 49,050,592.16$

Check it is a maximum:

$R''(x) = 400(e^{-0.003x}\cdot(-0.003)(1-0.003x)$

$+ e^{-0.003x}\cdot(-0.003))$

$= 400e^{-0.003x}(-0.006-0.000009x)$

Check $R''(\frac{1}{0.003}) < 0$ so it is a local max, must be absolute since only one crit. point

3. (6pts) The demand for bicycles at a sporting-goods store is given by $p = 500 - 20\ln x$, $0 \le x \le 1000$. If the bicycles cost the store \$350 each, how should they be priced to maximize profit? (Use the second-derivative test to check it's a maximum). What is the maximum profit?

$P(x) = R(x) - C(x)$

$= xp - C(x)$

$= x(500 - 20\ln x) - 350x$

$= 150x - 20x\ln x$

$P'(x) = 150 - 20(\ln x + x\cdot\frac{1}{x})$

$= 130 - 20\ln x$

$130 - 20\ln x = 0$

$\ln x = \frac{130}{20} = 6.5 \quad x = e^{6.5} \approx 665.142$

$P''(x) = -\frac{20}{x}, \quad P''(e^{6.5}) = -\frac{20}{e^{6.5}} < 0$

P has a local max. at $x = e^{6.5}$
- it must be an absolute max. since there is only one critical point.

Price: $p = 500 - 20\ln e^{6.5}$

$= 500 - 20\cdot 6.5 = \$370$

Max. profit $= 150\cdot e^{6.5} - 20e^{6.5}\underbrace{\ln e^{6.5}}_{6.5}$

$= 150e^{6.5} - 130e^{6.5} = \$13,302.83$

4. (7pts) The price-demand equation for T-shirts at a discount store is $p + 3x = 15$, where x thousand T-shirts are sold, $0 \le x \le 4$. Use elasticity of demand to solve the following problems.

a) If the current price of \$10.50 is decreased by 8%, by approximately what percentage will demand increase? Will revenue increase or decrease?

b) If the current price of \$6.00 is increased by 5%, by approximately what percentage will demand decrease? Will revenue increase or decrease?

$x = 5 - \frac{1}{3}p = f(p)$

$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p(-\frac{1}{3})}{5 - \frac{1}{3}p} \cdot \frac{3}{3} = \frac{p}{15-p}$

[- rel. rate of chg. of demand = $E(p)\cdot$ rel. rate of chg. of price]

a) $E(10.50) = \frac{10.50}{15 - 10.50} = \frac{10.50}{4.50} = 2.333$

$2.333 \cdot 8\% = 18.667$

A price decrease of 8% causes a demand increase of 18.667%

Revenue decreases since $E(10.50) > 1$

b) $E(6) = \frac{6}{15-6} = \frac{6}{9} = 0.667$

$0.667 \cdot 5\% = 3.333\%$

A price increase of 5% causes a demand decrease of 3.333%. Revenue increases since $E(6) < 1$