Fall '06/MAT 220/Worksheet 6 **Name:** Solution *Show all your work.*

1. (4pts) Evaluate without using the calculator:

$\log_3 81 =$ 4 $\quad\quad \log_4 \frac{1}{16} =$ −2 $\quad\quad \log_{34} 1 =$ 0 $\quad\quad \log_a \sqrt[6]{a^5} =$ $\frac{5}{6}$

$3^{?} = 81 \quad\quad 4^{?} = \frac{1}{16} = \frac{1}{4^2} \quad\quad 34^{?} = 1 \quad\quad a^{?} = \sqrt[6]{a^5} = a^{\frac{5}{6}}$

2. (4pts) Solve the following equations by turning them to exponential form:

$\log_3 x = \frac{1}{2}$ $\quad\quad \log_x 7 = 4$

$3^{\frac{1}{2}} = x$

$x = \sqrt{3} \approx 1.732$

$x^4 = 7$

$x = \sqrt[4]{7} \approx 1.627$

3. (6pts) Suppose \$2,000 is invested into an account paying 6.21% compounded monthly.
a) How much is in the account in two and a half years?
b) How long will it take until the account is worth \$5,000?

a) $A = 2000\left(1 + \frac{0.0621}{12}\right)^{12 \cdot 2.5}$

$= 2000(1.005175)^{30}$

$\approx \$2334.96$

$\left(A = P\left(1 + \frac{r}{m}\right)^{mt}\right)$

b) $5000 = 2000\left(1 + \frac{0.0621}{12}\right)^{12t} \quad | \div 2000$

$2.5 = 1.005175^{12t} \quad | \ln$

$\ln 2.5 = \ln 1.005175^{12t}$

$\ln 2.5 = 12t \ln 1.005175$

$t = \frac{\ln 2.5}{12 \ln 1.005175} = 14.793$ years

4. (4pts) At what nominal rate compounded continuously must money be invested to double in 3 years?

$A = Pe^{rt}$

$2P = Pe^{r \cdot 3} \quad | \div P$

$2 = e^{3r} \quad | \ln$

$\ln 2 = \ln e^{3r}$

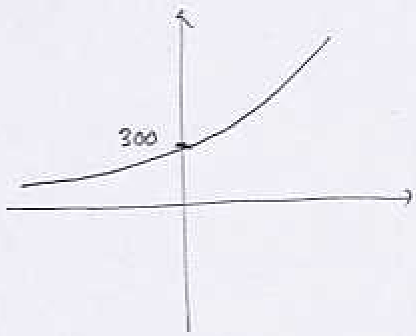
$\ln 2 = 3r$

$r = \frac{\ln 2}{3} = 0.23105$

rate = 23.105%

5. (6pts) The US population is 300 million today and grows according to the model $P = P_0 e^{rt}$ (pg. 313) at the continuous compound rate or growth of 0.85%.
a) Write the function that describes the US population (in millions) t years from now. Then graph the function here.
b) How long until the US population reaches 400 million?

a) $P = 300\, e^{0.0085t}$



b) $400 = 300\, e^{0.0085t} \quad | \div 300$

$\frac{4}{3} = e^{0.0085t} \quad | \ln$

$\ln \frac{4}{3} = 0.0085t$

$t = \frac{\ln \frac{4}{3}}{0.0085} \approx 33.845$ years

6. (6pts) Radioactive substances decay according to the law $Q = Q_0 e^{rt}$ (pg. 313). A radioactive isotope of carbon, carbon 14, takes 5600 years to decay to half the original amount (that is, the *half-life* of C14 is 5600 years).
a) Find the continuous compound rate r of decay for C14.
b) How long until a sample of C14 decays to one-third of the original amount?

a) $\frac{1}{2} Q_0 = Q_0 e^{r \cdot 5600} \quad | \div Q_0$

$\frac{1}{2} = e^{r\, 5600} \quad | \ln$

$\ln \frac{1}{2} = 5600 r$

$r = \frac{\ln \frac{1}{2}}{5600} \approx -0.0001238$

b) $\frac{1}{3} Q_0 = Q_0\, e^{-0.0001238t} \quad | \div Q_0$

$\frac{1}{3} = e^{-0.0001238t} \quad | \ln$

$\ln \frac{1}{3} = -0.0001238t$

$t = \frac{\ln \frac{1}{3}}{-0.0001238} \approx 8875.79$ years