Fall '06/MAT 220/Worksheet 4 **Name:** Solution *Show all your work.*

1. (10pts) Let $f(x) = (x^2 - 4)^2$.
a) Find the x-intercepts and the y-intercept.
b) Find the intervals of increase and decrease and find the local extrema.
c) Find the intervals where the function is concave up/down and find the inflection points.
d) Sketch a nice graph of the function that takes into account everything you found in a)-c).

a) x-int.: $(x^2-4)^2 = 0$ y-int: $f(0) = 16$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

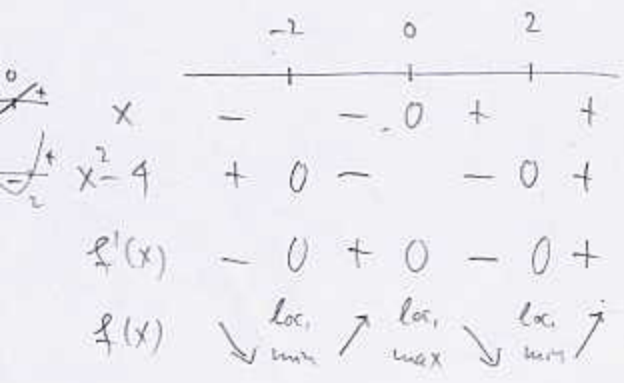
b) $f'(x) = 2(x^2-4)2x$

$= 4x(x^2-4)$

$4x(x^2-4) = 0$

$x = 0, \quad x^2 - 4 = 0$

$x = \pm 2$



x	f(x)
-2	0
0	16
2	0

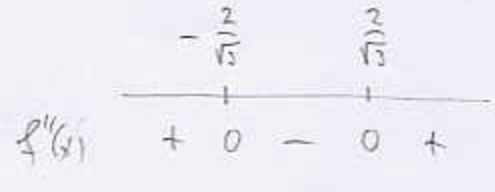
c) $f'(x) = 4x^3 - 16x$

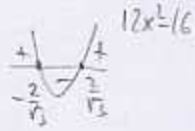
$f''(x) = 12x^2 - 16$

$12x^2 - 16 = 0$

$x^2 = \frac{16}{12} = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}} \approx \pm 1.155$

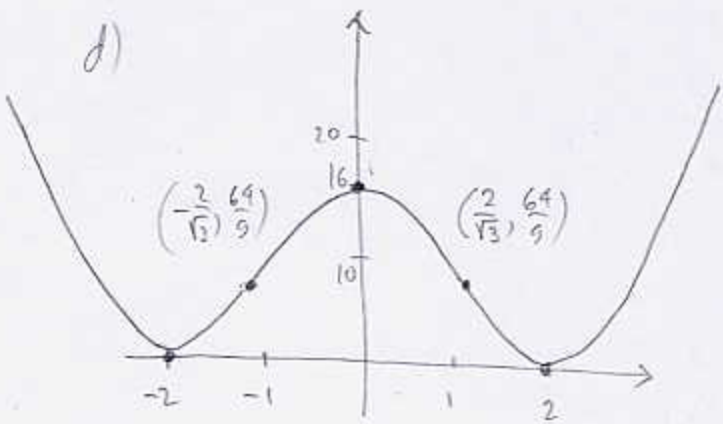




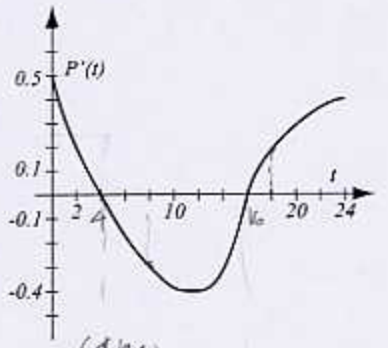
x	f(x)
$-\frac{2}{\sqrt{3}}$	$\frac{64}{9} \approx 7.111$
$-\frac{2}{\sqrt{3}}$	$\frac{64}{9} \approx 7.111$

$\left(\frac{4}{3} - 4\right)^2 = \left(-\frac{8}{3}\right)^2 = \frac{64}{9}$

d)



2. (10pts) The graph in the figure gives the rate of change $P'(t)$ of the price of electricity $P(t)$ over 24 months. (Units for $P(t)$ are cents/kWh).
a) During which time was the price of electricity increasing? Decreasing?
b) What is $P'(8)$? $P'(18)$? Explain in words the meaning of these numbers.
c) Sketch a possible graph for $P(t)$. Try to make it as accurate as you can. You may assume $P(0) = 9$.

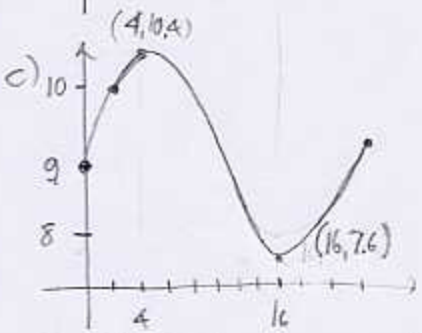


a) Increasing where $P'(t) > 0$, on $(0,4)$ and $(16,24)$
Decreasing where $P'(t) < 0$, on $(4,16)$

b) $P'(8) = -0.3$ At time $t=8$, price of electricity is decreasing at rate 0.3 c/mo.

$P'(18) = 0.2$ At time $t=18$, price of electricity is increasing at rate 0.2 c/mo

c)



t	0	2	4	6	8	10	12	14	16	18	20	22	24
est. $P(t)$	9	10	10.4	10.4	10.2	9.6	8.8	8	7.6	7.6	8	8.6	9.4
$P'(t)$	0.5	0.2	0	-0.1	-0.3	-0.4	-0.4	-0.2	0	0.2	0.3	0.4	

3. (10pts) A company estimates that it will sell $N(x)$ units of a product after spending x thousand on advertising, as given by $N(x) = -0.25x^4 + 13x^3 - 180x^2 + 10000$, $15 \le x \le 24$.
a) When is the rate of change of sales increasing? Decreasing?
b) What is the point of diminishing returns and the maximum rate of change of sales?
c) Graph N and N' on the same coordinate system. Where is the inflection point on the graph of N?

a) $N'(x) = -x^3 + 39x^2 - 360x$

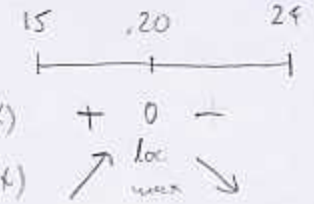
$N''(x) = -3x^2 + 78x - 360$

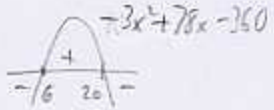
$-3x^2 + 78x - 360 = 0 \quad | \div -3$

$x^2 - 26x + 120 = 0$

$x = \dfrac{26 \pm \sqrt{676 - 4 \cdot 1 \cdot 120}}{2}$

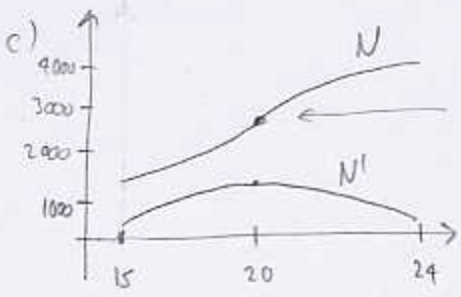
$= \dfrac{26 \pm \sqrt{196}}{2} = \dfrac{26 \pm 14}{2} = 20, 6$





b) The point of diminishing returns is $x = 20$
Max. rate of change of sales is $N'(20) = 400$ units / thou. dollars

c)



inflection point for $x = 20$