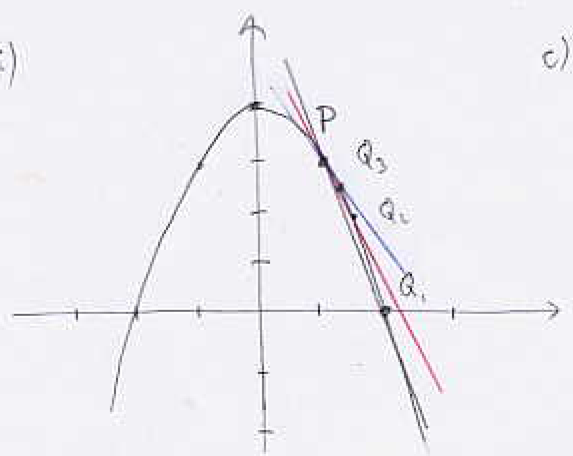
Fall '06/MAT 220/Worksheet 3 **Name:** Solution *Show all your work.*

1. (9pts) This problem is about finding the tangent line to the curve $f(x) = 4 - x^2$ at the point $P = (1, 3)$.
a) Draw the graph of the function on the interval $[-3, 3]$.
b) Draw three secant lines passing through P and a point Q on the graph, where Q is $(2, f(2))$, $(1.5, f(1.5))$, $(1.25, f(1.25))$ respectively.
c) Find slopes of the lines PQ for each of the three choices and estimate the slope of the tangent line from this information.
d) Use a limit to algebraically find the slope of the tangent line at $(1, 3)$.
e) Write the equation of the desired tangent line.

a)



b)

x	$4-x^2$
2	0
1.5	1.75
1.25	2.4375

c) $m_{PQ_1} = \dfrac{0-3}{2-1} = -3$

$m_{PQ_2} = \dfrac{1.75-3}{1.5-1} = \dfrac{-1.25}{0.5} = -2.5$

$m_{PQ_3} = \dfrac{2.4375-3}{1.25-1} = \dfrac{-0.5625}{0.25} = -2.25$

estimate of slope of tangent line: -2

d) $$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{4-(1+h)^2-3}{h} =$$

$$= \lim_{h\to 0} \frac{1-(1+2h+h^2)}{h} = \lim_{h\to 0} \frac{-2h-h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2-h)}{\cancel{h}} = -2$$

e) $y - 3 = -2(x-1)$

$y = -2x + 5$

2. (4pts) Use a limit (i.e. "four-step process") to find $f'(x)$ of the function $f(x) = \sqrt{3+2x}$.

$$\lim_{h\to 0} \frac{\sqrt{3+2(x+h)} - \sqrt{3+2x}}{h} \cdot \frac{\sqrt{3+2(x+h)} + \sqrt{3+2x}}{\sqrt{3+2(x+h)} + \sqrt{3+2x}} = \lim_{h\to 0} \frac{3+2(x+h) - (3+2x)}{h\left(\sqrt{3+2(x+h)} + \sqrt{3+2x}\right)}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{3+2x+2h} + \sqrt{3+2x}\right)} = \frac{2}{\sqrt{3+2x} + \sqrt{3+2x}} = \frac{2}{2\sqrt{3+2x}} = \frac{1}{\sqrt{3+2x}}$$

3. (10pts) Use differentiation rules to find the derivates:

a) $\frac{d}{dx}(4x^3 - 7x^2 + 13x - 6) =$ $12x^2 - 14x + 13$

b) $\frac{d}{du}\left(\frac{3}{u} - \frac{7}{u^6}\right) =$ $\frac{d}{du}\left(3u^{-1} - 7u^{-6}\right) = -3u^{-2} + 42u^{-7} = -\frac{3}{u^2} + \frac{42}{u^7}$

c) $\frac{d}{dt}\left(\sqrt{t} + \frac{3}{\sqrt{t}}\right) =$ $\frac{d}{dt}\left(t^{\frac{1}{2}} + 3t^{-\frac{1}{2}}\right) = \frac{1}{2}t^{-\frac{1}{2}} + 3\cdot\left(-\frac{1}{2}\right)t^{-\frac{3}{2}} = \frac{1}{2\sqrt{t}} - \frac{3}{2\sqrt{t^3}}$

d) $(2\sqrt[4]{x^9} + 5x^{3.4})' =$ $\left(2x^{\frac{9}{4}} + 5x^{3.4}\right)' = 2\cdot\frac{9}{4}x^{\frac{5}{4}} + 5\cdot 3.4x^{2.4} = \frac{9}{2}x^{\frac{5}{4}} + 17x^{2.4}$

4. (7pts) Let $g(x) = 2x^3 - 9x^2 - 60x + 17$.
a) Algebraically find the values of x where the graph of g has a horizontal tangent line.
b) Use your calculator to draw the graph of g (on paper!). Verify your answer from a) on the graph.

a) $g'(x) = 6x^2 - 18x - 60$

Find when $g'(x) = 0$

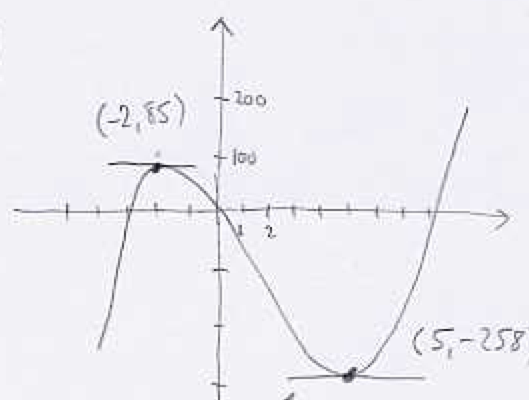
$6x^2 - 18x - 60 = 0 \quad | \div 6$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, -2$

b)



It does have horizontal tangent lines at $x = -2, 5$.