Fall '06/MAT 220/Worksheet 2 **Name:** Solution *Show all your work.*

1. (7pts) The polynomial $f(x) = x^4 - 2x^3 - 5x^2 - x - 7$ is given.
a) Use Theorem 2 in 2.1 to find an interval on the x-axis that must contain all the zeros of this polynomial
b) Graph the function on this interval (yes, on paper!).
c) Find all the zeros to three decimal places.
d) Does this polynomial have the maximal number of zeros possible for a fourth-degree polynomial?

a) $\max\{|-2|, |-5|, |-1|, |-7|\} = 7$

$|r| < 8$

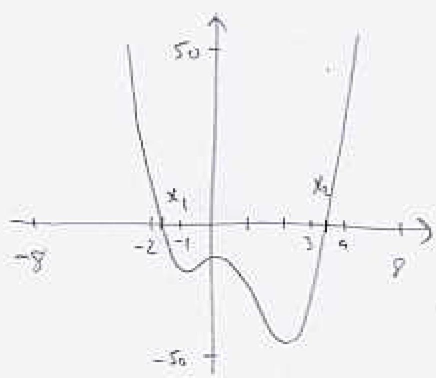
$-8 < r < 8$

c) $x_1 = -1.770$

$x_2 = 3.610$

d) No, a fourth-degree polynomial can have up to four zeroes.

b)



2. (5pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

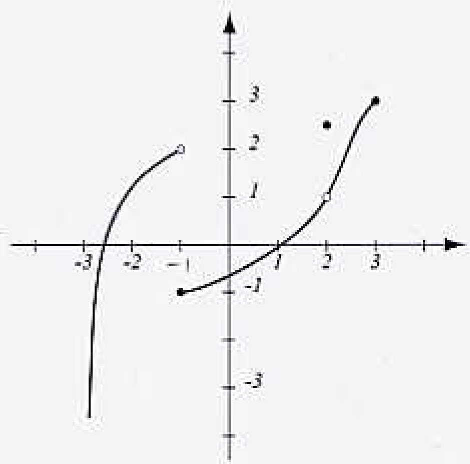
$\lim_{x \to -1^-} f(x) =$ 2

$\lim_{x \to -1^+} f(x) =$ -1

$\lim_{x \to -1} f(x) =$ d.n.e. since one-sided limits are different

$\lim_{x \to 2} f(x) =$ 1

$f(2) =$ 2.5



3. (10pts) The rational function $f(x) = \dfrac{2x^2+6}{x^2-4x+3}$ is given.

a) Find all the x- and y-intercepts for the graph.

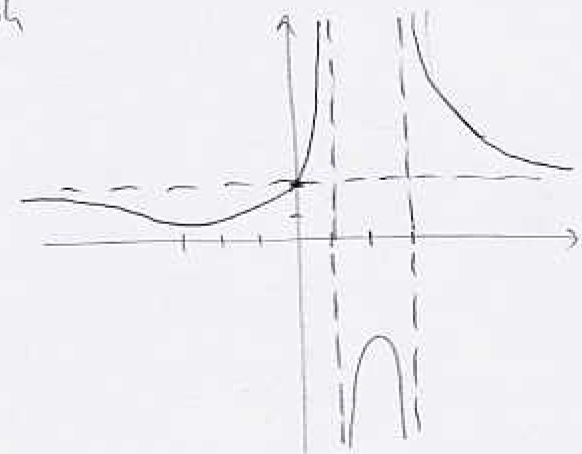
b) Find the vertical asymptotes of the graph.

c) Sketch a graph of the function on paper.

d) Find algebraically if the graph has a horizontal asymptote. If so, display it on your graph.

e) $\lim_{x\to 3^+} f(x) = \infty$ } from graph

f) $\lim_{x\to -\infty} f(x) = 2$



a) $f(0) = \frac{6}{3} = 2$ y-int.

$2x^2+6=0$

$x^2=-3$

no sol., so no x-int.

b) $x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=1, 3$

Vertical asymptotes:

$x=1, \ x=3$

d) $f(x)$ behaves like $\frac{2x^2}{x^2}=2$ for large $|x|$

so $y=2$ is horizontal asymptote

4. (8pts) Consider the limit $\lim_{x\to 5} \dfrac{x-5}{\sqrt{x}-\sqrt{5}}$.

a) Use your calculator to make a table of values. Estimate the limit to three decimal places.

b) Find the limit algebraically and compare your answer to a).

eval. at 5 gives $\frac{0}{0}$

a)

x	$f(x)$
4.9	4.449662
4.99	4.469899
4.999	4.471912
4.9999	4.472113
4.99999	4.472134

x	$f(x)$
5.1	4.494386
5.01	4.474371
5.001	4.472360
5.0001	4.472158
5.00001	4.472138

It appears limit is 4.472

b) $\lim_{x\to 5} \frac{x-5}{\sqrt{x}-\sqrt{5}} \cdot \frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}} =$

$= \lim_{x\to 5} \frac{(x-5)(\sqrt{x}+\sqrt{5})}{(\sqrt{x})^2-(\sqrt{5})^2}$

$= \lim_{x\to 5} \frac{\cancel{(x-5)}(\sqrt{x}+\sqrt{5})}{\cancel{x-5}}$

$= \lim_{x\to 5}(\sqrt{x}+\sqrt{5}) = \sqrt{5}+\sqrt{5} = 2\sqrt{5} \approx 4.472136$

agrees with a)