Fall '06/MAT 220/Exam 1　　**Name:**　　*Show all your work.*

1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x \to 3^-} f(x) =$

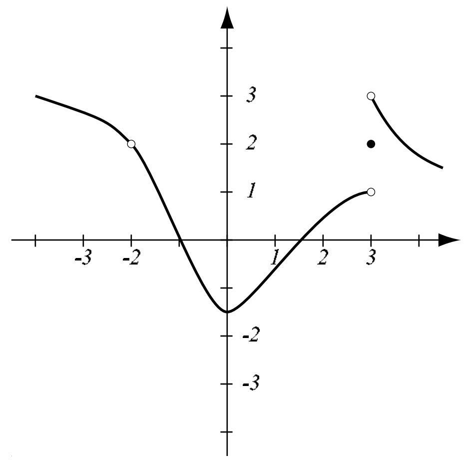
$\lim_{x \to 3^+} f(x) =$

$\lim_{x \to 3} f(x) =$

$\lim_{x \to -2} f(x) =$

$f(-2) =$

Is f continuous at $x = -2$?
Why or why not?



2. (6pts) Algebraically find the following limits:

a) $\lim_{x \to 2} \dfrac{x+1}{x^3 - 4x^2 + x + 7} =$

b) $\lim_{x \to 5} \dfrac{x-5}{x^2 - 3x - 10} =$

3. (6pts) The polynomial $f(x) = x^3 - 5x^2 + 4x + 3$ is given.
a) Graph the function on paper.
b) Find all the turning points to three decimal places.
c) Does this polynomial have the maximal number of turning points possible for a third-degree polynomial?

4. (7pts) The accounting department at a company that produces a certain kind of window finds that it has fixed costs (at 0 output) of \$4,000 per day and total costs of \$12,000 per day when 160 windows are produced.
a) Assuming total cost per day $C(x)$ is a linear function, find an equation for $C(x)$.
b) What is the predicted cost of manufacturing 100 windows per day?
c) Sketch the graph for $0 \leq x \leq 200$.

5. (12pts) A company produces LCD televisions. Analysts at its financial department have found that the price-demand and cost functions are given by

$$p(x) = 950 - 40x \qquad C(x) = 900 + 400x \qquad 0 \leq x \leq 10$$

where x is in millions, p in dollars and C in millions of dollars.
a) Write the revenue function $R(x)$ and the profit function $P(x)$.
b) Graph the profit function on paper for $0 \leq x \leq 10$.
c) Algebraically find the level of production that gives the highest profit. What is the highest profit possible?
d) Algebraically find the break-even points.
e) For what range of production levels is this enterprise profitable?

6. (7pts) Let $f(x) = \dfrac{x+5}{x^2-9}$.

a) Find algebraically if the graph of this rational function has a horizontal asymptote.
b) Use your calculator to make a table of values for f that will help you find $\lim_{x\to\infty} f(x)$. Then estimate this limit.
c) How do your answers and a) and b) compare?

7. (5pts) Use a sign chart to solve the inequality: $\dfrac{3x+2}{4-2x} < 0$.

Bonus. (5pts) Draw the graph of a function that is continuous and defined at all points except $x = 1$ and $x = 4$ and satisfies all of the following:

$\lim_{x\to 1^+} f(x) = \infty$

$\lim_{x\to 1^-} f(x) = \infty$

$\lim_{x\to 4^-} f(x) = -2$

$\lim_{x\to 4^+} f(x) = 3$

Fall '06/MAT 220/Exam 2 Name: *Show all your work.*

1. (17pts) Find the following derivatives, simplifying where possible.

$(7x^3 - x^2 + 13x - 7)' =$

$$\frac{d}{dx}\left(\frac{1}{x} - 3\sqrt[5]{x^9}\right) =$$

$$\frac{d}{du}(\sqrt{u}(u^2 + u - 1)) =$$

$$\frac{d}{dx}\frac{x+1}{3x^2 - 6x + 17} =$$

$$\frac{d}{dt}(t^{16} + t^8)^{\frac{3}{2}} =$$

2. (8pts) Let $f(x) = x^2 + 5x$.
a) Use a limit to find $f'(x)$.
b) Now find $f'(x)$ using differentiation rules and compare to your answer in a).
c) Find the equation of the tangent line to the graph of f at the point $(3, f(3))$.

3. (6pts) The total cost of producing x electric guitars is $C(x) = 1000 + 100x - 0.25x^2$.
a) Find the exact cost of producing the 51st guitar.
b) Use marginal cost to approximate the cost of producing the 51st guitar. Is this approximation accurate?

4. (7pts) Let $g(x) = x^3 - 6x^2 + 12x + 8$.
a) Algebraically find the values of x where the graph of g has a horizontal tangent line.
b) Use your calculator to draw the graph of g (on paper!). Verify your answer from a) on the graph.

5. (7pts) The total monthly cost of producing x ski jackets is given by $C(x) = 24x + 21,900$. The monthly price-demand function is given by $p(x) = 200 - 0.2x$, $0 \le x \le 1000$.
a) Form the profit function for this situation.
b) Find the profit and the marginal profit for $x = 500$. Interpret the meaning of the marginal profit.
c) Someone from the company asks you whether they should increase production from level $x = 500$ for the purpose of increasing profit. What do you tell them?

6. (5pts) Find the derivative, and simplify.

$$\frac{d}{dx}((x^2+1)^3(x^3+2)^4) =$$

Bonus. (5pts) A falling Bösendorfer[1] is $5t^2$ meters away from the release point after t seconds of travel time.

a) Find the average velocities of the Bösendorfer over the three time intervals that start with $t = 1$ seconds and last, respectively, 0.5, 0.05 and 0.005 seconds.

b) Find the instantaneous velocity of the piano at time $t = 1$. (Note: no need for limit here.)

c) What is the connection between the numbers in a) and the number in b)?

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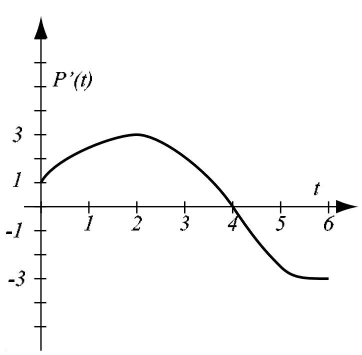
[1] a brand of piano

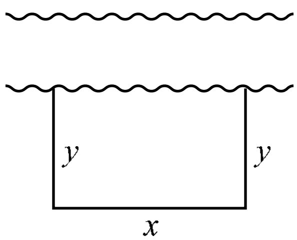
1. (13pts) Let $f(x) = x^4 - 18x^2 + 10$.
a) Find the intervals of increase and decrease and find the local extrema.
b) Find the intervals where the function is concave up/down and find the inflection points.
c) Use your calculator to find the x-intercepts to accuracy three decimal points. Helpful tip: Here they are symmetric about the y-axis. Then find the y-intercept.
d) Sketch a nice graph of the function that takes into account everything you found in a)-c).

2. (8pts) Let $f(x) = x^3 - 12x^2 + 36x + 7$. Find the absolute extremes of this function (and where they occur) on the closed intervals $[-1, 4]$. Use your calculator to graph the function (on paper!) and verify your findinds.

3. (11pts) The graph in the figure gives the rate of change $P'(t)$ of the price of a barrel of oil over 6 months. The following questions are about the price of oil $P(t)$ and its graph. (Units for $P(t)$ are dollars.)
a) During which time was the price of oil increasing? Decreasing?
b) During which time is the graph of P concave up? Concave down?
c) What is $P'(2)$? $P'(5)$? State their units and explain in words the meaning of these numbers.
d) Sketch a possible graph for $P(t)$. Make it as accurate as you can. Assume $P(0) = 45$.



4. (10pts) The "Talking Heads" will use a fence to enclose a rectangular area along a river (picture). The side along the river is not fenced. If the rectangle is to have area 2000m^2, what should be its dimensions in order to minimize the length of the fence used? What is the minimal length of the fence? (When solving this problem, state the domain of your variable and use the second derivative test to ensure that your solution is indeed a minimum.)



5. (8pts) Let f be a rational function whose domain is all real x except for $x = -3$ and $x = 3$. Below you can see some information for f and the sign charts for f' and f''. Answer the following:

a) Where does f have vertical asymptotes, if any?
b) Complete the sign chart of f' by indicating where f is increasing or decreasing.
c) Complete the sign chart of f'' by indicating where f is concave up or down.
d) Where does f have a local extreme, if any?
e) Where does f have an inflection point, if any?
f) Use information from a)-e) to draw the graph of f.

x	-3	-1	0	0.35	3	4
$f(x)$	ND	0	1.11	1.4	ND	0

$\lim_{x\to\infty} f(x) = 2, \lim_{x\to-\infty} f(x) = 2$

		-3		3	
f'	+	ND	+	ND	+
f		ND		ND	

		-3		0.35		3	
f''	+	ND	-	0	+	ND	-
f		ND				ND	

Bonus. (5pts) Try to come up with a formula for a rational function that fits the above problem. (Hint: what should be in the numerator? In the denominator? How do you get the correct limit at ∞?)

Fall '06/MAT 220/Exam 4 **Name:** *Show all your work.*

1. (9pts) Find the following derivatives (simplify where possible):

a) $\dfrac{d}{dx}\dfrac{x^{36}}{e^x} =$

b) $\dfrac{d}{dt}\, e^{3t}\sqrt{t} =$

c) $\dfrac{d}{dx}\ \ln(x^2 + x - 5)^5 =$

2. (3pts) For the following, use the chain rule to find $\dfrac{dy}{dx}$ and express in terms of x.

$y = ue^u$ and $u = x^2 - x$,

3. (6pts) Suppose \$3,000 is invested into an account paying 4.5% compounded quarterly.
a) How much is in the account in three years?
b) How long will it take until the account is worth \$4,000?

4. (4pts) The U.S population was approximately 151 million in 1950 and is approximately 300 million today. If population grew according to the model $P = P_0 e^{rt}$, what is the continuous compound rate of growth for those 56 years?

5. (8pts) The demand for sneakers at a sporting-goods store is given by $p = 150 - 18 \ln x$, $0 \leq x \leq 250$. If the sneakers cost the store \$42 each, how should they be priced to maximize profit? (Use the second-derivative test to check it's a maximum). What is the maximum profit?

6. (12pts) Let $f(x) = x^2 e^x$.
a) Find the intervals of increase and decrease and find the local extrema.
b) Find the intervals where the function is concave up/down and find the inflection points.
c) Find the x-intercepts and the y-intercept.
d) Sketch a nice graph of the function that takes into account everything you found in a)-c).
e) Does the graph have a horizontal asymptote? If so, what is it?

7. (8pts) The price-demand equation for a sandwich at a fast food restaurant is $x + 400p = 2000$, where x sandwiches are sold, $0 \leq p \leq 5$.
a) Find elasticity of demand as a function of price.
b) If the current price of \$2.25 is increased by 6%, by approximately what percentage will demand decrease? Will revenue increase or decrease?
c) Find the values of p where demand is elastic? Inelastic?

Bonus. (5pts) If you draw a the tangent line to the graph of $\ln x$ at $x = 3$, it appears that it will pass through the origin. Verify whether this is true by a computation.

1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x \to -1^-} f(x) =$

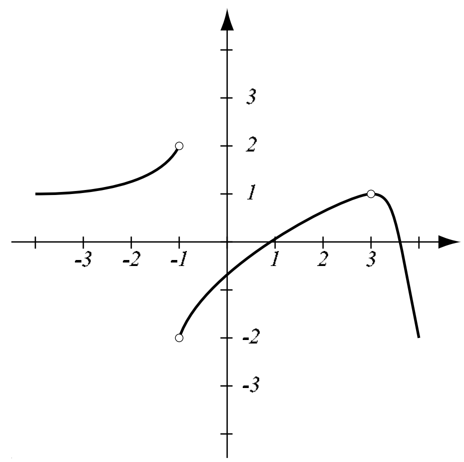
$\lim_{x \to -1^+} f(x) =$

$\lim_{x \to -1} f(x) =$

Is f continuous at $x = -1$?
Why or why not?

$\lim_{x \to 3} f(x) =$

$f(3) =$



2. (6pts) Consider the limit $\lim_{x \to 5} \frac{x^2 + x - 30}{x - 5}$.
a) Use your calculator to make a table of values. Estimate the limit to three decimal places.
b) Find the limit algebraically and compare your answer to a).

3. (12pts) Find the following derivatives, simplifying where possible.

$$(7x^3 - \frac{1}{x^2} + 18)' =$$

$$\frac{d}{dx}\frac{x^2 - 3x + 4}{x + 1} =$$

$$\frac{d}{du}(\sqrt{u^3 - 1} \cdot \ln u) =$$

$$\frac{d}{dt}t^{3.5}e^{7t} =$$

4. (3pts) For the following, use the chain rule to find $\frac{dy}{dx}$ and express in terms of x.

$y = \ln u$ and $u = \ln(x^2 - 5)$,

5. (6pts) Find the equation of the tangent line to the graph of $f(x) = x^2 - 3x + 7$ at the point $(1, f(1))$. Sketch the graph of the function and the tangent line.

6. (11pts) Let $f(x) = (x - 3)e^x$.
a) Find the intervals of increase and decrease and find the local extrema.
b) Find the intervals where the function is concave up/down and find the inflection points.
c) Find the x-intercepts and the y-intercept.
d) Sketch a nice graph of the function that takes into account everything you found in a)-c).
e) Does the graph have a horizontal asymptote? If so, what is it?

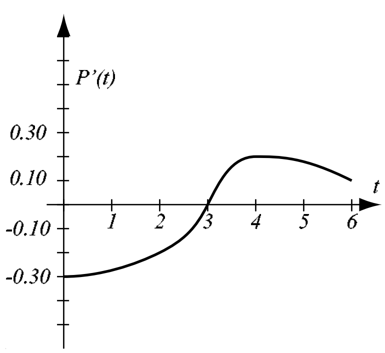
7. (6pts) Let $f(x) = 2x^3 - 15x^2 - 84x + 17$. Find the absolute extremes of this function (and where they occur) on the closed intervals $[4, 10]$.

8. (6pts) The graph in the figure gives the rate of change $P'(t)$ of the price of a gallon of gas over 6 months. The following questions are about the price of gas $P(t)$ and its graph. (Units for $P(t)$ are dollars.)

a) During which time was the price of gas increasing? Decreasing?

b) What is $P'(2)$? State its units and explain in words the meaning of this number.

c) Sketch a possible graph for $P(t)$, taking into account what you found in a). Assume $P(0) = \$2.20$.



9. (8pts) The demand for a modem at an electronics store is given by $p = 175 - 18 \ln x$, $0 \leq x \leq 300$. How should they be priced to maximize revenue? (Use the second-derivative test to check it's a maximum). What is the maximum revenue?

10. (5pts) At a children's store, the price-demand equation for a car seat for infants is $x + 40p = 2400$, where x car seats are sold, $0 \leq x \leq 2400$.
a) Find elasticity of demand as a function of price.
b) If the current price of \$35 is decreased by 7%, by approximately what percentage will demand increase? Will revenue increase or decrease?

Bonus. (7pts) Use the same demand function $x + 40p = 2400$, as in the previous problem.
a) Write revenue as a function of price p.
b) Draw the graph of revenue as a function of price. How can you see from the graph for which prices the demand is elastic? Inelastic?
c) Assuming car seats cost the store \$19 per seat, write the profit as a function of p.
d) At what price does the store achieve maximum profit, and how much is the maximum profit?