Fall '06/MAT 220/Final Exam **Name:** Solution *Show all your work.*

1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -1^-} f(x) =$ 2

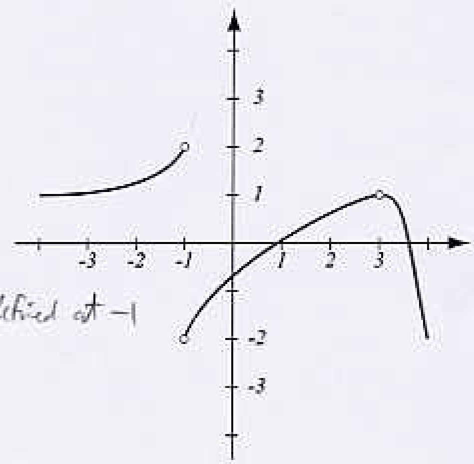
$\lim_{x\to -1^+} f(x) =$ -2

$\lim_{x\to -1} f(x) =$ d.n.e. since one-sided limits are different

Is f continuous at $x = -1$?
Why or why not? No, because f is not defined at -1

$\lim_{x\to 3} f(x) =$ 1

$f(3) =$ not defined



2. (6pts) Consider the limit $\lim_{x\to 5} \dfrac{x^2+x-30}{x-5}$.
a) Use your calculator to make a table of values. Estimate the limit to three decimal places.
b) Find the limit algebraically and compare your answer to a).

a)

x	$\frac{x^2+x-30}{x-5}$
5.1	11.1
5.01	11.01
5.001	11.001
5.0001	11.0001
4.9	10.9
4.99	10.99
4.999	10.999
4.9999	10.9999

$$\lim_{x\to 5} \frac{x^2+x-30}{x-5}$$

$$= \lim_{x\to 5} \frac{(x+6)(x-5)}{x-5}$$

$$= \lim_{x\to 5} (x+6)$$

$$= 11$$

3. (12pts) Find the following derivatives, simplifying where possible.

$(7x^3 - \frac{1}{x^2} + 18)' =$ $21x^2 - (-2)x^{-3} = 21x^2 + \frac{2}{x^3}$

x^{-2}

$\frac{d}{dx}\frac{x^2-3x+4}{x+1} =$ $\frac{(2x-3)(x+1)-(x^2-3x+4)\cdot 1}{(x+1)^2} = \frac{2x^2-x-3-x^2+3x-4}{(x+1)^2} = \frac{x^2+2x-7}{(x+1)^2}$

$\frac{d}{du}(\sqrt{u^3-1}\cdot \ln u) =$ $\frac{1}{2\sqrt{u^3-1}}\cdot 3u^2 \ln u + \sqrt{u^3-1}\cdot\frac{1}{u} = \frac{3u^2 \ln u}{2\sqrt{u^3-1}} + \frac{\sqrt{u^3-1}}{u}$

$\frac{d}{dt}t^{3.5}e^{7t} =$ $3.5t^{2.5}e^{7t} + t^{3.5}e^{7t}\cdot 7 = e^{7t}(3.5t^{2.5} + 7e^{3.5})$

4. (3pts) For the following, use the chain rule to find $\frac{dy}{dx}$ and express in terms of x.

$y = \ln u$ and $u = \ln(x^2-5)$,

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}\cdot\frac{1}{x^2-5}\cdot 2x = \frac{2x}{\ln(x^2-5)(x^2-5}$

5. (6pts) Find the equation of the tangent line to the graph of $f(x) = x^2 - 3x + 7$ at the point $(1, f(1))$. Sketch the graph of the function and the tangent line.

$f'(x) = 2x - 3$

$f'(1) = -1$

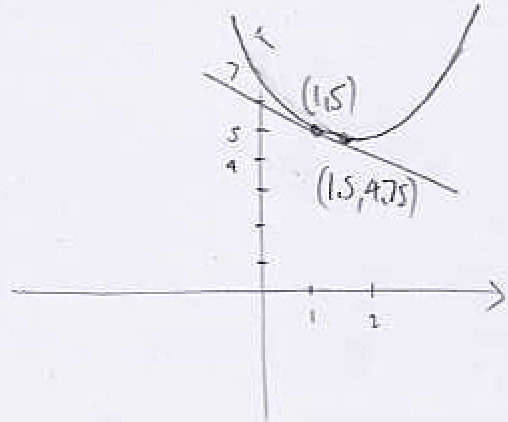
Eq. of tangent line

$f(1) = 5$

$y - 5 = -1(x-1)$

$y = -x + 1 + 5$

$y = -x + 6$



6. (11pts) Let $f(x) = (x-3)e^x$.

a) Find the intervals of increase and decrease and find the local extrema.

b) Find the intervals where the function is concave up/down and find the inflection points.

c) Find the x-intercepts and the y-intercept.

d) Sketch a nice graph of the function that takes into account everything you found in a)-c).

e) Does the graph have a horizontal asymptote? If so, what is it?

a) $f'(x) = 1 \cdot e^x + (x-3)e^x$

$= (1 + x - 3)e^x$

$= (x-2)\underbrace{e^x}_{>0}$

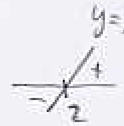
Critical pts: $x - 2 = 0$

$x = 2$

f' has same sign as $x-2$

		2	
f'	$-$	0	$+$
f	$\searrow$	loc. min	$\nearrow$

$y = x - 2$



b) $f''(x) = 1 \cdot e^x + (x-2)e^x = (1 + x - 2)e^x = (x-1)e^x$

$x - 1 = 0$

$x = 1$

f'' has same sign as $x - 1$

		1	
f''	$-$	0	$+$
f	CD	infl. pt.	CU

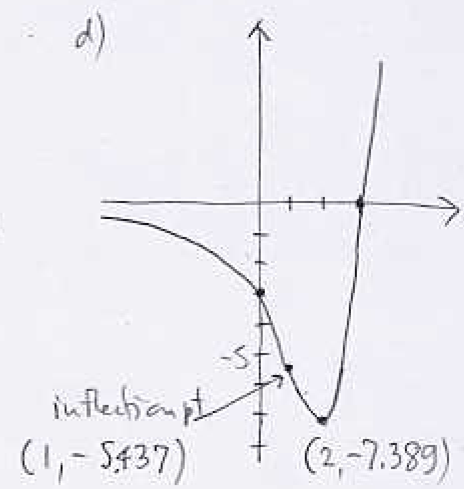
c) $(x-3)e^x = 0$

$x - 3 = 0$

$x = 3$ x-int

$f(0) = -3$ y-int

d)



e) y-axis is the horizontal asymptote

7. (6pts) Let $f(x) = 2x^3 - 15x^2 - 84x + 17$. Find the absolute extremes of this function (and where they occur) on the closed intervals $[4, 10]$.

$f'(x) = 6x^2 - 30x - 84$

$6x^2 - 30x - 84 = 0 \quad | \div 6$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7, -2$

critical pts,

only $x=7$ is in the interval

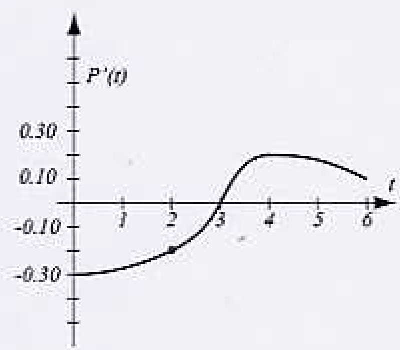
x	$f(x)$	
4	-431	
10	-323	abs. max,
7	-620	abs. min,

8. (6pts) The graph in the figure gives the rate of change $P'(t)$ of the price of a gallon of gas over 6 months. The following questions are about the price of gas $P(t)$ and its graph. (Units for $P(t)$ are dollars.)

a) During which time was the price of gas increasing? Decreasing?

b) What is $P'(2)$? State its units and explain in words the meaning of this number.

c) Sketch a possible graph for $P(t)$, taking into account what you found in a). Assume $P(0) = \$2.20$.

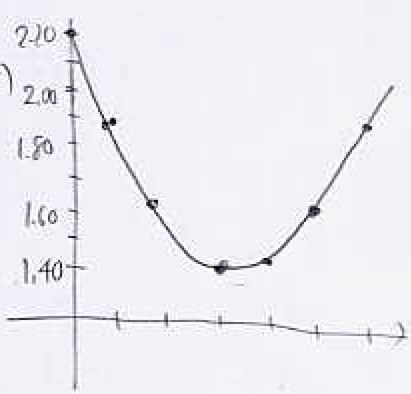


a) P is increasing when $P'(t) > 0$, on $(3,6)$
decreasing when $P'(t) < 0$, on $(0,3)$

b) $P'(2) = -0.20$ \$/month

Price of gas was decreasing at rate \$0.20/month

c)



t	0	1	2	3	4	5	6
$P(t)$	2.20	1.90	1.63	1.43	1.43	1.63	1.81
$P'(t)$	-0.30	-0.27	-0.20	0	0.20	0.18	

10000
9. (8pts) The demand for a modem at an electronics store is given by $p = 175 - 18\ln x$, $0 \le x \le \cancel{300}$. How should they be priced to maximize revenue? (Use the second-derivative test to check it's a maximum). What is the maximum revenue?

$R = xp = x(175 - 18\ln x)$

$R(x) = 175x - 18x\ln x$

$R'(x) = 175 - 18(1\cdot\ln x + x\cdot\frac{1}{x})$

$= 175 - 18\ln x - 18$

$= 157 - 18\ln x$

$157 - 18\ln x = 0$

$\ln x = \frac{157}{18} = 8.722$

$x = e^{8.722} = 6137.80$

$p = 175 - 18\cdot\ln e^{\frac{157}{18}} = 175 - 157 = 18$

$R = 110,480.46$

$R'(x) = -18\cdot\frac{1}{x}$

$R''(x) = -\frac{18}{e^{8.72}} < 0$, so a local max

Since it is the only critical point, it must be an abs. min,

10. (5pts) At a children's store, the price-demand equation for a car seat for infants is $x + 40p = 2400$, where x car seats are sold, $0 \le x \le 2400$.
a) Find elasticity of demand as a function of price.
b) If the current price of \$35 is decreased by 7%, by approximately what percentage will demand increase? Will revenue increase or decrease?

a) $E(p) = -\frac{p f'(p)}{f(p)} = -\frac{p(-40)}{2400-40p} = \frac{40p}{40(60-p)} = \frac{p}{60-p}$

$x = 2400 - 40p = f(p)$

b) $E(35) = \frac{35}{60-35} = \frac{35}{25} = 1.4 > 1$ demand is elastic, so price and revenue move in opposite directions – revenue will increase.

Demand increases by

$1.4\cdot 7\% = 9.8\%$

Bonus. (7pts) Use the same demand function $x + 40p = 2400$, as in the previous problem.
a) Write revenue as a function of price p.
b) Draw the graph of revenue as a function of price. How can you see from the graph for which prices the demand is elastic? Inelastic?
c) Assuming car seats cost the store \$19 per seat, write the profit as a function of p.
d) At what price does the store achieve maximum profit, and how much is the maximum profit?

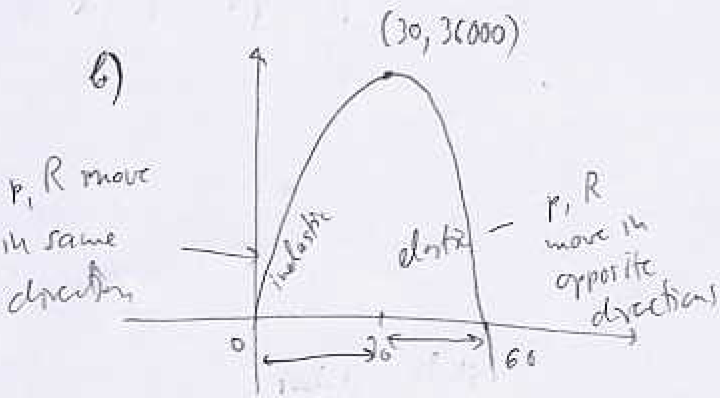
a) $x + 40p = 2400$

$x = 2400 - 40p$

$R = xp = (2400 - 40p)p$

$= -40p^2 + 2400p$

b)



c) $P = R - C = -40p^2 + 2400p - 19x$

$= -40p^2 + 2400p - 19(2400 - 40p)$

$= -40p^2 + 2400p - 45600 + 760p$

$= -40p^2 + 3160p - 45600$

Also a parabola

d) $P'(p) = -80p + 3160$

$-80p + 3160 = 0$

$p = \frac{3160}{80} = 39.50$

$P''(p) = -80 < 0$ so it is a local max

Max profit is: $P(39.50) = 16{,}810$