Fall '06/MAT 220/Exam 4 **Name:** Solution *Show all your work.*

1. (9pts) Find the following derivatives (simplify where possible):

a) $\frac{d}{dx}\frac{x^{36}}{e^x} = \frac{36x^{35}e^x - x^{36}e^x}{(e^x)^2} = \frac{x^{35}e^x(36-x)}{(e^x)^2} = \frac{x^{35}(36-x)}{e^x}$

b) $\frac{d}{dt}e^{3t}\sqrt{t} = e^{3t}\cdot 3\sqrt{t} + e^{3t}\frac{1}{2\sqrt{t}} = e^{3t}\left(3\sqrt{t} + \frac{1}{2\sqrt{t}}\right)$

c) $\frac{d}{dx}\ln(x^2+x-5)^5 = \frac{d}{dx}5\ln(x^2+x-5) = 5\cdot\frac{1}{x^2+x-5}\cdot(2x+1) = \frac{5(2x+1)}{x^2+x-5}$

2. (3pts) For the following, use the chain rule to find $\frac{dy}{dx}$ and express in terms of x.

$y = ue^u$ and $u = x^2 - x$,

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = (e^u + ue^u)(2x-1) = e^u(u+1)(2x-1)$$

$$= e^{x^2-x}(x^2-x+1)(2x-1)$$

3. (6pts) Suppose \$3,000 is invested into an account paying 4.5% compounded quarterly.
a) How much is in the account in three years?
b) How long will it take until the account is worth \$4,000?

a) $A = P\left(1+\frac{r}{m}\right)^{4t}$

$A = 3000\left(1+\frac{0.045}{4}\right)^{4\cdot 3}$

$= 3000(1.01125)^{12}$

$= 3431.02$

b) $4000 = 3000(1.01125)^{4t}$ $\quad |\div 3000$

$\frac{4}{3} = 1.01125^{4t}$ $\quad |\ln$

$\ln\frac{4}{3} = \ln 1.01125^{4t}$

$\ln\frac{4}{3} = 4t\ln 1.01125$

$t = \frac{\ln\frac{4}{3}}{4\ln 1.01125} = 6.429$ years

4. (4pts) The U.S population was approximately 151 million in 1950 and is approximately 300 million today. If population grew according to the model $P = P_0e^{rt}$, what is the continuous compound rate of growth for those 56 years?

$P = P_0 e^{rt}$

$300 = 151 e^{r \cdot 56}$

$\frac{300}{151} = e^{56r}$ | ln

$\ln \frac{300}{151} = \ln e^{56r}$

$56r = \ln \frac{300}{151}$

$r = \frac{\ln \frac{300}{151}}{56} \approx 0.01226$

$r = 1.226\%$

5. (8pts) The demand for sneakers at a sporting-goods store is given by $p = 150 - 18\ln x$, $0 \le x \le 250$. If the sneakers cost the store \$42 each, how should they be priced to maximize profit? (Use the second-derivative test to check it's a maximum). What is the maximum profit?

$P(x) = px - C(x)$

$= (150 - 18\ln x)x - 42x$

$= 108x - 18x\ln x$

$P'(x) = 108 - 18(1 \cdot \ln x + x \cdot \frac{1}{x})$

$= 108 - 18\ln x - 18$

$= 90 - 18\ln x$

$90 - 18\ln x = 0$

$\ln x = \frac{90}{18} = 5$

$e^{\ln x} = e^5$

$x = e^5 \approx 148.41$

$P''(x) = -\frac{18}{x}$

$P''(e^5) = -\frac{18}{e^5} < 0$

So it is a local max. Since there is only one critical point, it must be an absolute max.

Price is

$p = 150 - 18\ln e^5 = 150 - 18 \cdot 5 = \boxed{\$60}$

Max. profit is

$P(e^5) = 108e^5 - 18e^5 \underbrace{\ln e^5}_{5}$

$\boxed{= \$2671.44}$

6. (12pts) Let $f(x) = x^2 e^x$.

a) Find the intervals of increase and decrease and find the local extrema.

b) Find the intervals where the function is concave up/down and find the inflection points.

c) Find the x-intercepts and the y-intercept.

d) Sketch a nice graph of the function that takes into account everything you found in a)-c).

e) Does the graph have a horizontal asymptote? If so, what is it?

a) $f'(x) = 2xe^x + x^2 e^x$

$= e^x(2x + x^2)$

$\underbrace{e^x}_{>0}(2x+x^2) = 0$

$x^2 + 2x = 0$

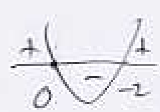
$x(x+2) = 0$

$x = 0, -2$

f' has sign of x^2+2x

		-2		0	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	loc. max	$\searrow$	loc. min	$\nearrow$

$y = x^2 + 2x$



b) $f''(x) = e^x(2x+x^2) + e^x(2+2x)$

$= e^x(x^2+4x+2)$

$\underbrace{e^x}_{>0}(x^2+4x+2) = 0$

$x^2+4x+2 = 0$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 1}}{2}$

$= \dfrac{-4 \pm \sqrt{8}}{2} = \dfrac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$

f'' has sign of x^2+4x+2

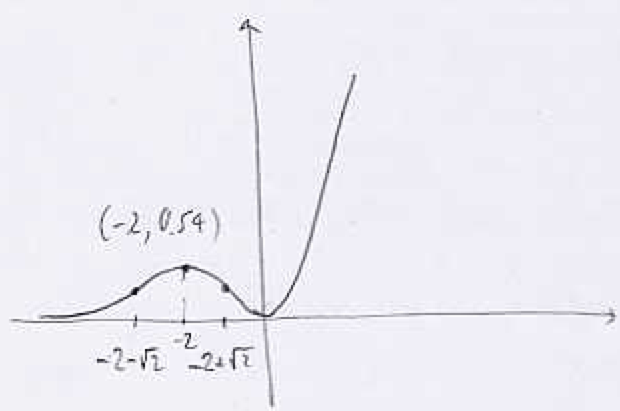
		$-2-\sqrt{2}$		$-2+\sqrt{2}$	
$f''(x)$	$+$	0	$-$	0	$+$
$f(x)$	CU	$\cdot$	CD	$\cdot$	CU

infl. pts.

c) $x^2 e^x = 0$ $\quad f(0) = 0$

$x = 0$ $\quad$ y-int

x-int



e) y-axis is horizontal asymptote.

7. (8pts) The price-demand equation for a sandwich at a fast food restaurant is $x + 400p = 2000$, where x sandwiches are sold, $0 \leq p \leq 5$.
a) Find elasticity of demand as a function of price.
b) If the current price of \$2.25 is increased by 6%, by approximately what percentage will demand decrease? Will revenue increase or decrease?
c) Find the values of p where demand is elastic? Inelastic?

a) $x = 2000 - 400p$

$2000 - 400p > 0$

$2000 > 400p$

$5 > p$

$E = -\frac{p f'(p)}{f(p)} = -\frac{p(-400)}{2000-400p} = \frac{400p}{2000-400p} = \frac{p}{5-p}$

b) $E(2.25) = \frac{2.25}{5-2.25} = \frac{2.25}{2.75} = 0.818 < 1$ inelastic demand

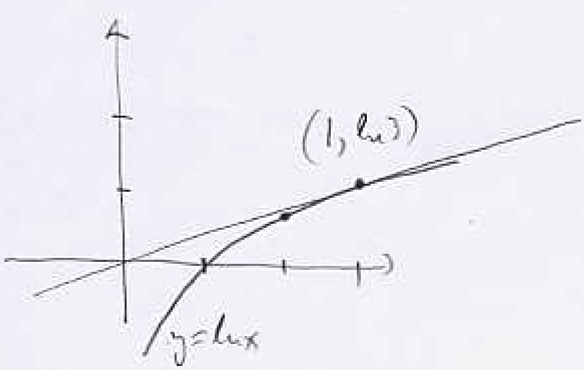
Demand decreases $0.818 \cdot 6\% = 4.9\%$. Revenue increases.

c) Solve $\frac{p}{5-p} < 1 \quad | \cdot 5-p$ $\quad p < 5-p$ $\quad$ Inelastic for $0 < p < 2.5$

$0 < p < 5$ $\quad 2p < 5$ $\quad$ Elastic for $2.5 < p < 0$

$p < 2.5$

Bonus. (5pts) If you draw a the tangent line to the graph of $\ln x$ at $x = 3$, it appears that it will pass through the origin. Verify whether this is true by a computation.



$(\ln x)' = \frac{1}{x}$

Slope of tan. line at $x=3$ is $\frac{1}{3}$

Eq. of tan. line:

$y - \ln 3 = \frac{1}{3}(x-1)$

$y = \frac{1}{3}x - \frac{1}{3} + \ln 3$

When $x = 0$, $y = \ln 3 - \frac{1}{3} = 0.098$, not 0.

Doesn't pass through origin.

---

[0]Total points: 50