Fall '06/MAT 220/Exam 3 Name: Solution *Show all your work.*

1. (13pts) Let $f(x) = x^4 - 18x^2 + 10$.

a) Find the intervals of increase and decrease and find the local extrema.

b) Find the intervals where the function is concave up/down and find the inflection points.

c) Use your calculator to find the x-intercepts to accuracy three decimal points. Then find the y-intercept.

d) Sketch a nice graph of the function that takes into account everything you found in a)-c).

a) $f'(x) = 4x^3 - 36x$

$4x^3 - 36x = 0$

$4x(x^2-9) = 0$

$x = 0$ or $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

		-3		0		3	
x	$-$		$-$	0	$+$		$+$
x^2-9	$+$	0	$-$		$-$	0	$+$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\searrow$	loc. min	$\nearrow$	loc. max	$\searrow$	loc. min	$\nearrow$

b) $f''(x) = 12x^2 - 36$

$12x^2 - 36 = 0 \quad \div 12$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

		$-\sqrt{3}$		$\sqrt{3}$	
f''	$+$		$-$		$+$
f	CU	$\uparrow$	CD	$\uparrow$	CU

inflection pts

c) $x_1 = -4.174$

$x_2 = -0.758$

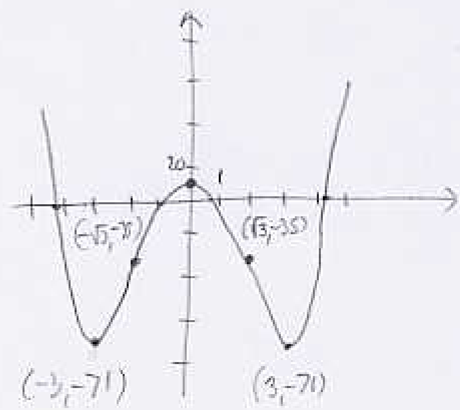
$x_3 = 0.758$

$x_4 = 4.174$

y-int: $f(0) = 10$

d)

x	$f(x)$
-3	-71
0	10
3	-71
$-\sqrt{3}$	-35
$\sqrt{3}$	35



2. (8pts) Let $f(x) = x^3 - 12x^2 + 36x + 7$. Find the absolute extremes of this function (and where they occur) on the closed intervals $[-1, 4]$. Use your calculator to graph the function (on paper!) and verify your findings.

$f'(x) = 3x^2 - 24x + 36$

$3x^2 - 24x + 36 = 0 \quad | \div 3$

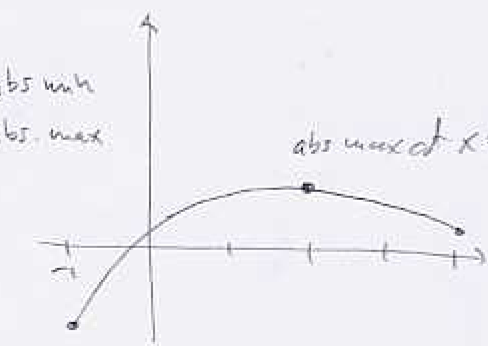
$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

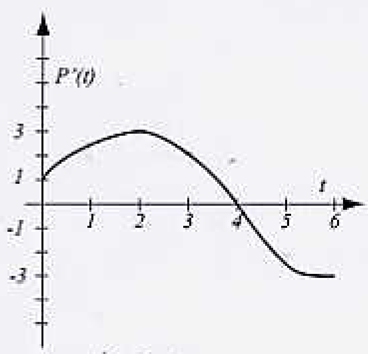
$x = 2, 6$

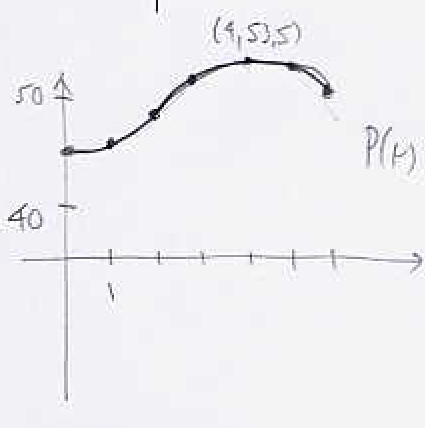
Test values on $[-1, 4]$

x	$f(x)$	
-1	-42	abs min
4	39	abs. max
2	23	



3. (11pts) The graph in the figure gives the rate of change $P'(t)$ of the price of a barrel of oil over 6 months. The following questions are about the price of oil $P(t)$ and its graph. (Units for $P(t)$ are dollars.)
a) During which time was the price of oil increasing? Decreasing?
b) During which time is the graph of P concave up? Concave down?
c) What is $P'(2)$? $P'(5)$? State their units and explain in words the meaning of these numbers.
d) Sketch a possible graph for $P(t)$. Make it as accurate as you can. Assume $P(0) = 45$.





a) Price was decreasing on $(4, 6)$ (where $P' < 0$)
increasing on $(0, 4)$ (where $P' > 0$)

b) P is concave up on $(0, 2)$ (where P' is increasing)
concave down on $(2, 6)$ (where P' is decreasing)

c) $P'(2) = 3$ \$/month
At time $t=2$, price is increasing at rate \$3/month
$P'(5) = -2.5$
At time $t=3$ price is decreasing at rate \$2.5/month

d)

t	0	1	2	3	4	5	6
$\approx P(t)$	45	46	48.5	51.5	53.5	53.5	51
$P'(t)$	1	2.5	3	2	0	-2.5	

5. (8pts) Let f be a rational function whose domain is all real x except for $x = -3$ and $x = 3$. Below you can see some information for f and the sign charts for f' and f''. Answer the following:

a) Where does f have vertical asymptotes, if any? at $x=-3$ and $x=3$

b) Complete the sign chart of f' by indicating where f is increasing or decreasing.

c) Complete the sign chart of f'' by indicating where f is concave up or down.

d) Where does f have a local extreme, if any? No extremes, always increasing

e) Where does f have an inflection point, if any? Inflection point at $x = 0.35$

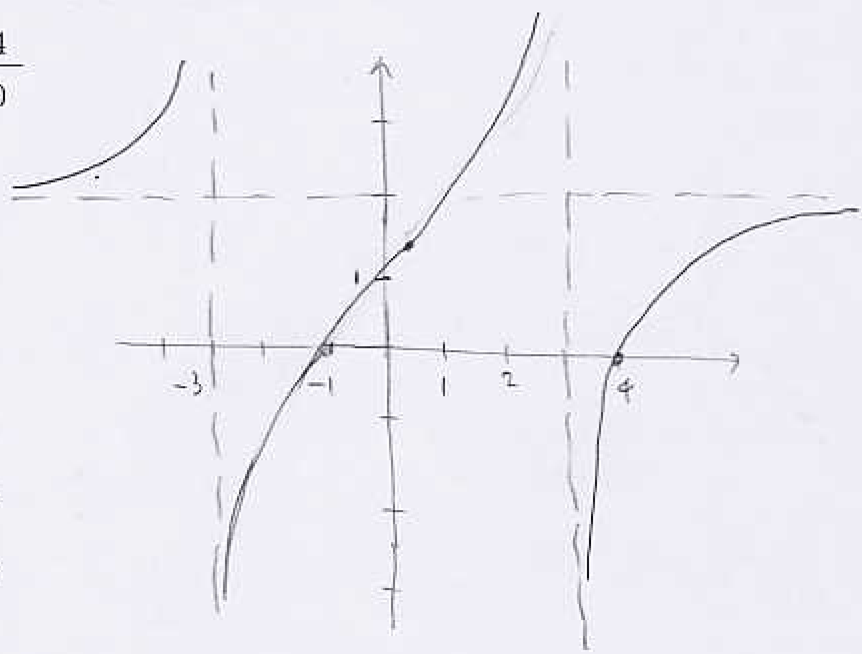
f) Use information from a)-e) to draw the graph of f.

x	-3	-1	0	0.35	3	4
$f(x)$	ND	0	1.11	1.4	ND	0

$\lim_{x\to\infty} f(x) = 2$, $\lim_{x\to-\infty} f(x) = 2$

		-3		3	
f'	+	ND	+	ND	+
f	↗	ND	↗	ND	↗

		-3		0.35		3	
f''	+	ND	-	0	+	ND	-
f	CU	ND	CD	infl. pt.	CU	ND	CD



Bonus. (5pts) Try to come up with a formula for a rational function that fits the above problem. (Hint: what should be in the numerator? In the denominator? How do you get the correct limit at ∞?)

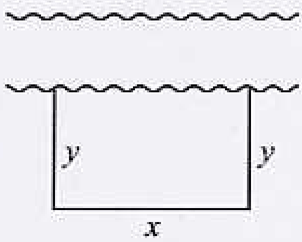
x-Intercepts are -1 and 4 so numerator has to contain $(x+1)$ and $(x-4)$

Not defined at $x=-3$ and $x=3$ so denominator has to contain $(x-3)$ and $(x+3)$

$y=2$ is horizontal asymptote, so numerator has to contain an extra 2

$$f(x) = \frac{2(x+1)(x-4)}{(x-3)(x+3)}$$

4. (10pts) The "Talking Heads" will use a fence to enclose a rectangular area along a river (picture). The side along the river is not fenced. If the rectangle is to have area 2000m^2, what should be its dimensions in order to minimize the length of the fence used? What is the minimal length of the fence? (When solving this problem, state the domain of your variable and use the second derivative test to ensure that your solution is indeed a minimum.)



$xy = 2000 \qquad y = \frac{2000}{x}$

$L = x + 2y = x + \frac{4000}{x}$ Minimize for $x > 0$

$L' = 1 + 4000(-1)x^{-2} = 1 - \frac{4000}{x^2}$

$L'' = 8000x^{-3} = \frac{8000}{x^3}$

Critical points,

$1 - \frac{4000}{x^2} = 0$

$1 = \frac{4000}{x^2}$

$x^2 = 4000$

$x = \pm\sqrt{4000}$

$x \approx 63.246$ m

(only $\sqrt{4000}$ valid)

$L''(\sqrt{4000}) = \frac{8000}{\sqrt{4000}^3} > 0$ so it is a local min.

Since $\sqrt{4000}$ is the only critical point, L has an absolute maximum at $x = \sqrt{4000}$.

Dimensions: $y = \frac{2000}{\sqrt{4000}} \approx 31.623$

Length of fence $= L(\sqrt{4000}) \approx 126.491$