Fall '06/MAT 220/Exam 2 **Name:** Solution *Show all your work.*

1. (17pts) Find the following derivatives, simplifying where possible.

$(7x^3 - x^2 + 13x - 7)' = 21x^2 - 2x + 13$

$$\frac{d}{dx}\left(\frac{1}{x} - 3\sqrt[5]{x^9}\right) = \frac{d}{dx}\left(x^{-1} - 3x^{\frac{9}{5}}\right) = -x^{-2} - \frac{27}{5}x^{\frac{4}{5}} = -\frac{1}{x^2} - \frac{27}{5}\sqrt[5]{x^4}$$

$$\frac{d}{du}(\overbrace{\sqrt{u}}^{u^{\frac{1}{2}}}(u^2 + u - 1)) = \frac{d}{du}\left(u^{\frac{5}{2}} + u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) = \frac{5}{2}u^{\frac{3}{2}} + \frac{3}{2}u^{\frac{1}{2}} - \frac{1}{2}u^{-\frac{1}{2}}$$

[or: (product rule)] $= \frac{1}{2\sqrt{u}}(u^2+u-1) + \sqrt{u}(2u+1) = \underbrace{\frac{u^2}{2\sqrt{u}}}_{=\frac{1}{2}u\sqrt{u}} + \underbrace{\frac{u}{2\sqrt{u}}}_{\frac{1}{2}\sqrt{u}} - \frac{1}{2\sqrt{u}} + 2u\sqrt{u} + \sqrt{u} = \frac{5}{2}u\sqrt{u} + \frac{3}{2}\sqrt{u} - \frac{1}{2\sqrt{u}}$

$$\frac{d}{dx}\frac{x+1}{3x^2 - 6x + 17} = \frac{1\cdot(3x^2-6x+17) - (x+1)(6x-6)}{(3x^2-6x+17)^2} = \frac{3x^2-6x+17-(6x^2-6)}{(3x^2-6x+17)^2}$$

$$= \frac{-3x^2-6x+23}{(3x^2-6x+17)^2}$$

$$\frac{d}{dt}(t^{16} + t^8)^{\frac{3}{2}} = \frac{3}{2}(t^{16}+t^8)^{\frac{1}{2}}(\underbrace{16t^{15}+8t^7}_{8(2t^{15}+t^7)}) = 12(t^{16}+t^8)^{\frac{1}{2}}(2t^{15}+t^7)$$

2. (8pts) Let $f(x) = x^2 + 5x$.
a) Use a limit to find $f'(x)$.
b) Now find $f'(x)$ using differentiation rules and compare to your answer in a).
c) Find the equation of the tangent line to the graph of f at the point $(3, f(3))$.

a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$

$= \lim_{h\to 0} \frac{(x+h)^2 + 5(x+h) - (x^2+5x)}{h}$

$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h - \cancel{x^2} - \cancel{5x}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(2x+h+5)}{\cancel{h}}$

$= \lim_{h\to 0} (2x+h+5) = 2x+5$

b) $f'(x) = 2x+5$ agrees with a)

c) $f'(3) = 2\cdot 3 + 5 = 11$

$f(3) = 9 + 15 = 24$

$y - 24 = 11(x-3)$

$y = 11x - 33 + 24$

$y = 11x - 9$

3. (6pts) The total cost of producing x electric guitars is $C(x) = 1000 + 100x - 0.25x^2$.
a) Find the exact cost of producing the 51st guitar.
b) Use marginal cost to approximate the cost of producing the 51st guitar. Is this approximation accurate?

a) $C(51) - C(50) =$

$= 1000 + 100\cdot 51 - 0.25\cdot 51^2$

$\quad - (1000 + 100\cdot 50 - 0.25\cdot 50^2)$

$= 5449.75 - 5375$

$= 74.75$

b) $C'(x) = 100 - 0.25\cdot 2x$

$= 100 - 0.5x$

$C'(50) = 100 - 0.5\cdot 50$

$= 75$

fairly accurate, off by only 0.25

4. (7pts) Let $g(x) = x^3 - 6x^2 + 12x + 8$.
a) Algebraically find the values of x where the graph of g has a horizontal tangent line.
b) Use your calculator to draw the graph of g (on paper!). Verify your answer from a) on the graph.

a) $g'(x) = 3x^2 - 12x + 12$

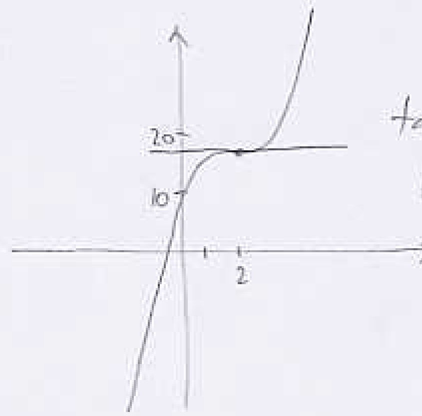
$3x^2 - 12x + 12 = 0$

$3(x^2 - 4x + 4) = 0$

$3(x-2)^2 = 0$

$x = 2$

b)



tan line is horizontal at $x=2$

5. (7pts) The total monthly cost of producing x ski jackets is given by $C(x) = 24x + 21,900$. The monthly price-demand function is given by $p(x) = 200 - 0.2x$, $0 \leq x \leq 1000$.
a) Form the profit function for this situation.
b) Find the profit and the marginal profit for $x = 500$. Interpret the meaning of the marginal profit.
c) Someone from the company asks you whether they should increase production from level $x = 500$ for the purpose of increasing profit. What do you tell them?

a) $R(x) = xp = x(200 - 0.2x)$
$= -0.2x^2 + 200x$

$P(x) = R(x) - C(x)$
$= -0.2x^2 + 200x - (24x + 21900)$
$= -0.2x^2 + 176x - 21900$

b) $P(500) = 16,100$

$P'(x) = -0.4x + 176$

$P'(500) = -0.4 \cdot 500 + 176$
$= -24$

$P'(500) = -24$ means that increasing production by one ski jacket reduces profit by about \$24.

c) Don't increase production! Since profit will drop by approx \$24 per jacket (at least for a small increase)

6. (5pts) Find the derivative, and simplify.

$$\frac{d}{dx}((x^2+1)^3(x^3+2)^4) = 3(x^2+1)^2 2x\,(x^3+2)^4 + (x^2+1)^3 4(x^3+2)^3 3x^2$$

$$= 6(x^2+1)^2(x^3+2)^3\left(x(x^3+2) + (x^2+1)\cdot 2x^2\right)$$

$$= 6x(x^2+1)^2(x^3+2)^3\left(x^3+2 + 2x(x^2+1)\right)$$

$$= 6x\,(x^2+1)^2(x^3+2)^3(3x^3+2x+2)$$

Bonus. (5pts) A falling Bösendorfer[1] is $5t^2$ meters away from the release point after t seconds of travel time.
a) Find the average velocities of the Bösendorfer over the three time intervals that start with $t = 1$ seconds and last, respectively, 0.5, 0.05 and 0.005 seconds.
b) Find the instantaneous velocity of the piano at time $t = 1$. (Note: no need for limit here.)
c) What is the connection between the numbers in a) and the number in b)?

a)

t	$5t^2$
1	5
1.5	11.25
1.05	5.5125
1.005	5.050125

$f(t) = 5t^2$ position

$$v_1 = \frac{11.25 - 5}{0.5} = 12.5$$

$$v_2 = \frac{5.5125 - 5}{0.05} = 10.25$$

$$v_3 = \frac{5.050125 - 5}{0.005} = 10.025$$

b) $v(t) = f'(t) = 10t$

$v(1) = 10$

c) The shorter the time interval, the closer average velocity is to instantaneous velocity.

[1] a brand of piano