Fall '06/MAT 220/Exam 1 **Name:** Solution *Show all your work.*

1. (7pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to 3^-} f(x) =$ 1

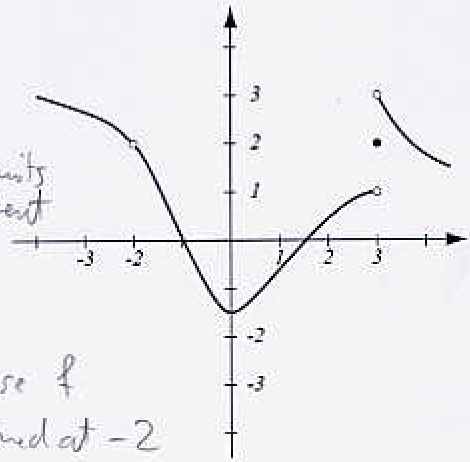
$\lim_{x\to 3^+} f(x) =$ 3

$\lim_{x\to 3} f(x) =$ d.n.e. since one-sided limits are different

$\lim_{x\to -2} f(x) =$ 2

$f(-2) =$ not defined

Is f continuous at $x = -2$? Why or why not? no, because f is not defined at -2



2. (6pts) Algebraically find the following limits:

a) $\lim_{x\to 2} \dfrac{x+1}{x^3-4x^2+x+7} =$

$$= \frac{2+1}{8-16+2+7} = \frac{3}{1} = 3$$

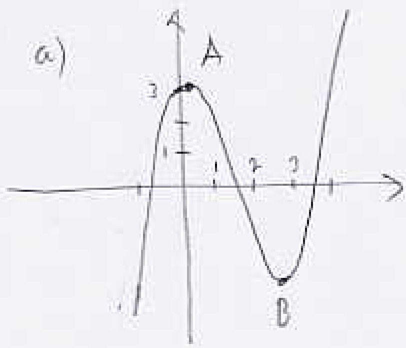
b) $\lim_{x\to 5} \dfrac{x-5}{x^2-3x-10} =$ $\left[\dfrac{5-5}{25-15-10} = \dfrac{0}{0} \right]$

$$= \lim_{x\to 5} \frac{\cancel{x-5}}{(x+2)\cancel{(x-5)}}$$

$$= \lim_{x\to 5} \frac{1}{x+2} = \frac{1}{7}$$

3. (6pts) The polynomial $f(x) = x^3 - 5x^2 + 4x + 3$ is given.
a) Graph the function on paper.
b) Find all the turning points to three decimal places.
c) Does this polynomial have the maximal number of turning points possible for a third-degree polynomial?

a)



b) Turning points

$A = (0.465, 3.879)$

$B = (2.869, -3.065)$

c) Yes, a third-degree polynomial has at most two turning points

4. (7pts) The accounting department at a company that produces a certain kind of window finds that it has fixed costs (at 0 output) of \$4,000 per day and total costs of \$12,000 per day when 160 windows are produced.
a) Assuming total cost per day $C(x)$ is a linear function, find an equation for $C(x)$.
b) What is the predicted cost of manufacturing 100 windows per day?
c) Sketch the graph for $0 \le x \le 200$.

a) Find line containing points: $(0, 4000)$

$(160, 12000)$

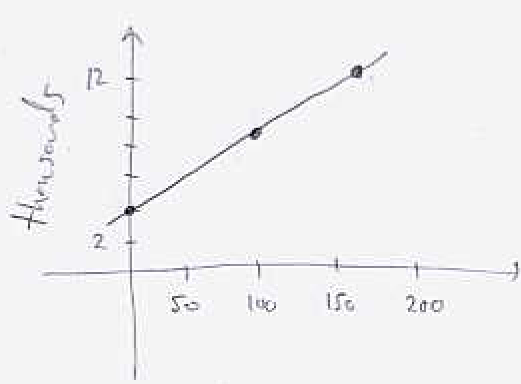
$$m = \frac{12000 - 4000}{160 - 0} = 50$$

$$y - 4000 = 50(x - 0)$$

$$C(x) = 50x + 4000$$

b) $C(100) = 50 \cdot 100 + 4000$

$= 9000$



5. (12pts) A company produces LCD televisions. Analysts at its financial department have found that the price-demand and cost functions are given by

$$p(x) = 950 - 40x \qquad C(x) = 900 + 400x \qquad 0 \le x \le 10$$

where x is in millions, p in dollars and C in millions of dollars.
a) Write the revenue function $R(x)$ and the profit function $P(x)$.
b) Graph the profit function on paper for $0 \le x \le 10$.
c) Algebraically find the level of production that gives the highest profit. What is the highest profit possible?
d) Algebraically find the break-even points.
e) For what range of production levels is this enterprise profitable?

a) $R(x) = x p(x)$
$= -40x^2 + 950x$

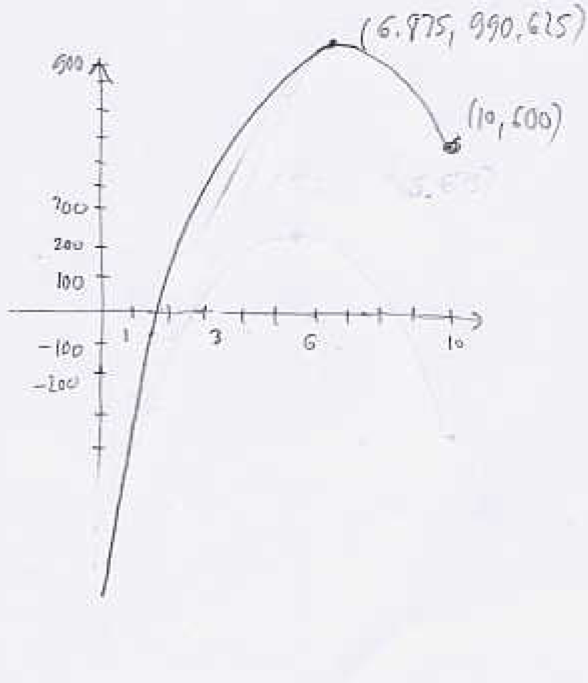
$P(x) = R(x) - C(x)$
$= -40x^2 + 950x - (900 + 400x)$
$= -40x^2 + 550x - 900$

c) $h = -\dfrac{550}{2\cdot(-40)} = \dfrac{550}{80} = 6.875$

$k = 990.625$

6,875,000 LCD TVS produced
max profit is 990,625,000

b)



d) $P(x) = 0$

$-40x^2 + 550x - 900 = 0$
$40x^2 - 450x + 900 = 0$

$$x = \frac{550 \pm \sqrt{550^2 - 4\cdot 40\cdot 900}}{2\cdot 40}$$
$$= \frac{550 \pm \sqrt{158500}}{80} = 11.852,\ 1.898$$

e) $1.898 \le x \le 10$
For production levels btw
1,898,000 and 10,000,000

6. (7pts) Let $f(x) = \dfrac{x+5}{x^2-9}$.

a) Find algebraically if the graph of this rational function has a horizontal asymptote.
b) Use your calculator to make a table of values for f that will help you find $\lim_{x\to\infty} f(x)$. Then estimate this limit.
c) How do your answers and a) and b) compare?

a) $f(x)$ behaves like $\dfrac{x}{x^2} = \dfrac{1}{x}$

$\dfrac{1}{x} \to 0$ when $x \to \infty$

so $y=0$ is the horizontal asymptote

b)

x	$f(x)$
100	0.011
1000	0.001005
10,000	0.0001
100,000	0.00001
1,000,000	0.000001

$\lim_{x\to\infty} f(x) = 0$ based on table

c) $\lim_{x\to\infty} f(x) = 0$ is same as saying $y=0$ is the horizontal asymptote

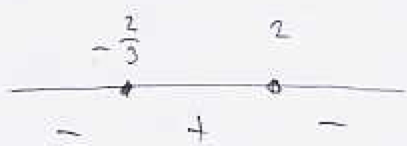
7. (5pts) Use a sign chart to solve the inequality: $\dfrac{3x+2}{4-2x} < 0$.

$3x+2=0$

$x = -\dfrac{2}{3}$

$4-2x=0$

$x=2$



x in $(-\infty, -\frac{2}{3}) \cup (2, \infty)$

x	$f(x)$	
−1	−1/6	−
0	2/4	+
3	11/−2	−

Bonus. (5pts) Draw the graph of a function that is continuous and defined at all points except $x = 1$ and $x = 4$ and satisfies all of the following:

$\lim_{x\to 1^+} f(x) = \infty$

$\lim_{x\to 1^-} f(x) = \infty$

$\lim_{x\to 4^-} f(x) = -2$

$\lim_{x\to 4^+} f(x) = 3$