

1. (20pts) A football team needs to select a captain from among four players: Steel, Griffin, Pollard and Driggs. Their preference rankings are as follows:

Number of voters:	12	5	8	1	10	3	2
Steel	1	1	2	3	4	2	2
Griffin	2	3	1	1	1	4	4
Pollard	3	2	3	4	2	1	3
Driggs	4	4	4	2	3	3	1

- a) Who wins the vote in a plurality election?
- b) Who wins the vote in a plurality election, followed by a runoff of the first two finishers?
- c) Who is the Condorcet winner, if any?
- d) Who is the winner using Borda's method? Perform the check on the sum of Borda points.
- e) In the Borda election, could the five voters who ranked Griffin last achieve a preferable outcome by voting strategically, assuming all the other members voted as shown?

a) Steel 17
Griffin 19 wins
Pollard 3
Driggs 2

b) Steel $17 + 3 + 2 = 22$ wins
Griffin $19 = 19$

d)
$$\begin{array}{r} \text{Steel} \quad 4 \cdot 17 + 3 \cdot 13 + 2 \cdot 1 + 1 \cdot 10 = 119 \\ \text{Griffin} \quad 4 \cdot 19 + 3 \cdot 12 + 2 \cdot 5 + 1 \cdot 5 = 127 \\ \text{Pollard} \quad 4 \cdot 3 + 3 \cdot 15 + 2 \cdot 22 + 1 \cdot 1 = 102 \\ \text{Driggs} \quad 4 \cdot 2 + 3 \cdot 1 + 2 \cdot 13 + 1 \cdot 25 = 62 \\ \hline 410 \end{array}$$

41 voters;
10 pts per ballot, $41 \cdot 10 = 410$

c) Steel 22 w
Griffin 19

Steel $17 + 8 + 1 = 26$ w
Pollard $10 + 3 + 2 = 15$

Steel $17 + 8 + 3 = 28$ w
Driggs $1 + 10 + 2 = 13$

Griffin $12 + 19 = 31$ w
Pollard $5 + 3 + 2 = 10$

Griffin $12 + 5 + 19 = 36$ w
Driggs $3 + 2 = 5$

Pollard $12 + 5 + 8 + 10 + 3 = 38$ w
Driggs $1 + 2 = 3$

Steel is the Condorcet winner

	Pts	From the 5 voters		
S	119	-9-6	104	+20=124
G	127	-3-2	122	+5=127
P	102	-12-4	86	+
D	62	-6-8	48	+

Pts if they vote
Steel 1st, Griffin 4th

Even if the 5 voters voted Steel 1st, Steel would not have enough to catch up to Griffin.