Spring '05/MAT 117/Exam 3b Name: Solution *Show all your work.*

$\frac{a}{b} = \frac{1-P(E)}{P(E)}$ $P(E) = \frac{b}{a+b}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

1. (2pts) What is the probability of picking up an expired jug of milk at the grocery store if 97% of them are have not expired on any given day?

$P(\text{expired}) = 1 - P(\text{not expired}) = 1 - 0.97 = 0.03$

2. (2pts) During the month of March, you saw a mouse in your kitchen on 13 days. What is the empirical probability of seeing a mouse in the kitchen on a random day in March?

empirical prob. $= \frac{13}{31}$

3. (7pts) A die is cast and a coin is tossed.
a) How many outcomes does this experiment have?
b) List the outcomes for which the number on the die contains the same letter as the result of the coin toss ("H" or "T").
c) What is the probability of the experiment resulting in a number on the die containing the letter representing the coin toss result?

a) 12 outcomes

1, H 1, T
2, H 2, T
⋮ ⋮
6, H 6, T

b) 2, T
3, T
3, H

c) $P = \frac{3}{12} = \frac{1}{4}$

4. (3pts) If a ball is drawn at random from a bag containing 3 black balls and 5 red balls, the odds against this ball being black are __5__ to __3__.

5. (3pts) The odds against finding a \$20 under Peter's couch are 133 to 2. What is the probability of finding \$20 under his couch?

$$P(\text{finding } \$20) = \frac{2}{133+2} = \frac{2}{135} = 0.0148$$

6. (4pts) A game is proposed to you: roll a die, and if you roll a 4 or a 5, you win. If the house odds on this game are 3 to 2, is this a fair bet? Hint: compute true odds against winning.

True odds against winning are 4 to 2

$\frac{3}{2} < \frac{4}{2}$ so it is an unfair bet,

7. (6pts) A bag contains one \$1,000 bill, three \$100 bills, five \$20 bills, ten \$5 bills and 1981 blank pieces of paper made from the same material as paper money. For a \$1 fee, you may draw without looking a bill from the bag and keep it. What is your expected value for the game?

outcomes: \$ 1000, \$ 100, \$ 20, \$ 5, \$ 0

(2000 bills

$$\begin{aligned}\text{expected win} &= 1000 \cdot P(\text{win } \$1000) \\ &+ 100 \cdot P(\text{win } \$100) \\ &+ 20 \cdot P(\text{win } \$20) \\ &+ 5 \cdot P(\text{win } \$5) \\ &+ 0 \cdot P(\text{win } \$0)\end{aligned}$$

expected value $= 0.725 - 1 = -0.275$

$$= 1000 \cdot \frac{1}{2000} + 100 \cdot \frac{3}{2000} + 20 \cdot \frac{5}{2000} + 5 \, \frac{10}{2000} + 0 \cdot \frac{1981}{2000}$$

$$= \frac{1000 + 300 + 100 + 50 + 0}{2000} = \frac{1450}{2000} = 0.725$$

8. (6pts) A driver fastens her seat belt 98% of the time and has her lights on 86% of the time. Assume that fastening the seat belt is independent from turning the lights on.
a) What is the probability that the driver has fastened her seatbelt and has her lights on?
b) What is the probability that the driver forgot to do at least one of the things mentioned?

a) $P\left(\begin{matrix}\text{fasten}\\ \text{seat belt}\end{matrix} \text{ and } \begin{matrix}\text{lights}\\ \text{on}\end{matrix}\right) = P\left(\begin{matrix}\text{fasten}\\ \text{seat-belt}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{lights}\\ \text{on}\end{matrix}\right)$

$= 0.98 \cdot 0.86 = 0.8428$

b) $P(\text{forgot at least one}) = P(\text{not (fasten and lights on)})$

$= 1 - 0.8428 = -0.1572$

9. (8pts) In a city with 77 restaurants, 27 have a salad bar, 43 have pizza on the menu and 19 have both. If a restaurant is randomly selected, what is the probability that
a) it has a salad bar or has pizza on the menu?
b) it does not have a salad bar or does not have pizza on the menu?

a) $P(\text{salad bar or pizza}) = P(\text{salad bar}) + P(\text{pizza}) - P(\text{salad bar and pizza})$

$= \frac{27}{77} + \frac{43}{77} - \frac{19}{77} = \frac{51}{77} = 0.6623$

b) $P(\text{no salad bar or no pizza}) = P(\text{no salad bar}) + P(\text{no pizza})$

$- P(\underbrace{\text{no salad bar and no pizza}}_{\text{not (salad bar or pizza)}})$

$= \frac{50}{77} + \frac{34}{77} - \left(1 - \frac{51}{77}\right)$

$= \frac{135}{77} - 1 = \frac{58}{77} = 0.7532$

Or: note that $P\left(\begin{matrix}\text{no}\\ \text{salad}\\ \text{bar}\end{matrix} \text{ or } \begin{matrix}\text{no}\\ \text{pizza}\end{matrix}\right) = P(\text{not (salad bar and pizza)}) = 1 - \frac{19}{77} = \frac{58}{77}$

10. (9pts) Two cards are drawn from a deck.
a) What is the probability that both cards are face cards?
b) What is the probability that the second card is a face card, if the first one was a king?
c) What is the probability that the second card is a king?

a) $P(\text{1st Face and 2nd Face}) = P(\text{1st Face}) \cdot P(\text{2nd Face} \mid \text{1st Face})$

$$= \frac{12}{52} \cdot \frac{11}{51} = 0.0498$$

b) $P(\text{2nd Face} \mid \text{1st King}) = \frac{11}{51} = 0.2157$

c) $P(\text{2nd King}) = \frac{4}{52} = \frac{1}{13} = 0.0769$

Bonus. (5pts) An old woman in Jakarta says: "It will rain today with 80% chance. If it rains today, it will rain tomorrow with 70% chance. If it doesn't rain today, then tomorrow's rain will come with 90% chance." What is the probability that it rains on exactly one of the days?

$$P\Big(\underbrace{\big(\text{rain today and no rain tomorrow}\big) \text{ or } \big(\text{no rain today and rain tomorrow}\big)}_{\text{mutually exclusive}}\Big)$$

$$= P(\text{rain today and no rain tomorrow}) + P(\text{no rain today and rain tomorrow})$$

$$= P(\text{rain today}) \cdot P(\text{no rain tomorrow} \mid \text{rain today}) + P(\text{no rain today})\, P(\text{rain tomorrow} \mid \text{no rain today})$$

$$= 0.8 \cdot 0.3 + 0.2 \cdot 0.9$$

$$= 0.24 + 0.18 = 0.42$$