

$$\frac{a}{b} = \frac{1-P(E)}{P(E)} \quad P(E) = \frac{b}{a+b} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(B|A) = \frac{n(A \text{ and } B)}{n(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

1. (2pts) During the month of December, you saw a Santa on 19 days. What is the empirical probability of seeing a Santa on a random day in December?

$$P(\text{see Santa}) = \frac{19}{31}$$

2. (2pts) What is the probability of picking up a stale bag of potato chips at the grocery store if 98% of them are fresh on any given day?

$$P(\text{bag stale}) = 1 - P(\text{bag fresh}) = 1 - 0.98 = 0.02$$

3. (7pts) A die is cast and a coin is tossed.

a) How many outcomes does this experiment have?

b) List the outcomes for which the number on the die shares a vowel with the result of the coin toss (e.g. "one" shares the vowel "e" with "heads").

c) What is the probability of the experiment resulting in a number on the die sharing a vowel with the result of the coin toss?

a) 1, H 1, T
2, H 2, T
⋮ ⋮
6, H 6, T

36 outcomes

b) 1, H
3, H
5, H
5, T
6, T

c) $P = \frac{5}{36}$

4. (3pts) If a ball is drawn at random from a bag containing 3 black balls and 5 red balls, the odds against this ball being black are 5 to 3.

5. (3pts) The odds against finding a mouse under Peter's couch are 25 to 2. What is the probability of finding a mouse under his couch?

$$P(\text{find mouse}) = \frac{2}{25+2} = \frac{2}{27} = 0.0741$$

6. (4pts) A game is proposed to you: roll a die, and if you roll a 5, you win. If the house odds on this game are 4 to 1, is this a fair bet? Hint: compute true odds against winning.

True odds: 5 to 1

$$\frac{4}{1} < \frac{5}{1} \quad \text{so not a fair bet.}$$

7. (6pts) A bag contains one \$1,000 bill, three \$100 bills, five \$20 bills, ten \$5 bills and 1981 blank pieces of paper made from the same material as paper money. For a \$1 fee, you may draw without looking a bill from the bag and keep it. What is your expected value for the game?

outcomes: 1000
100
20
5
0

$$\begin{aligned} \text{expected win} &= 1000 \cdot P(\text{win } \$1000) + 100 \cdot P(\text{win } \$100) \\ &\quad + 20 \cdot P(\text{win } \$20) + 5 \cdot P(\text{win } \$5) \\ &\quad + 0 \cdot P(\text{win } \$0) \\ &= 1000 \cdot \frac{1}{2000} + 100 \cdot \frac{3}{2000} + 20 \cdot \frac{5}{2000} + 5 \cdot \frac{10}{2000} + 0 \cdot \frac{1981}{2000} \\ &= \frac{1000 + 300 + 100 + 50 + 0}{2000} = \frac{1450}{2000} = 0.725 \end{aligned}$$

$$\text{expected value} = 0.725 - 1 = -0.275$$

8. (6pts) In a city with 77 restaurants, 27 have a salad bar, 43 have pizza on the menu and 19 have both. If a restaurant is randomly selected, what is the probability that

a) it has a salad bar or has pizza on the menu?

b) it neither has a salad bar nor has pizza on the menu?

$$\begin{aligned} \text{a) } P(\text{salad bar or pizza}) &= P(\text{salad bar}) + P(\text{pizza}) - P(\text{salad bar and pizza}) \\ &= \frac{27}{77} + \frac{43}{77} - \frac{19}{77} = \frac{51}{77} = 0.6623 \end{aligned}$$

$$\begin{aligned} \text{b) } P(\text{no salad bar and no pizza}) &= P(\text{not (salad bar or pizza)}) \\ &= 1 - \frac{51}{77} = \frac{26}{77} = 0.3377 \end{aligned}$$

9. (8pts) A driver fastens her seat belt 98% of the time and has her lights on 86% of the time. Assume that fastening the seat belt is independent from turning the lights on.

a) What is the probability that the driver has fastened her seatbelt and has her lights on?

b) What is the probability that the driver fastened her seatbelt and forgot to turn her lights on?

$$\begin{aligned} \text{a) } P(\text{seat belt fastened and lights on}) &= P(\text{seat belt fastened}) \cdot P(\text{lights on}) \\ &= 0.98 \cdot 0.86 = 0.8428 \end{aligned}$$

$$\begin{aligned} \text{b) } P(\text{seat belt fastened and lights not on}) &= P(\text{seat belt fastened}) \cdot P(\text{lights not on}) \\ &= 0.98 \cdot 0.14 = 0.1372 \end{aligned}$$

10. (9pts) Two cards are drawn from a deck.

a) What is the probability that both cards are spades?

b) What is the probability that the second card is a spade, if the first one was a heart?

c) What is the probability that the second card is a heart?

$$\begin{aligned} \text{a) } P(\text{1st spade and 2nd spade}) &= P(\text{1st spade}) \cdot P(\text{2nd spade} \mid \text{1st spade}) \\ &= \frac{13}{52} \cdot \frac{12}{51} = 0.0588 \end{aligned}$$

$$\text{b) } P(\text{2nd spade} \mid \text{1st heart}) = \frac{13}{51} = 0.2549$$

$$\begin{aligned} \text{c) } P(\text{2nd heart}) &= \frac{13}{52} \quad (\text{as though we draw just one card}) \\ &= 0.25 \end{aligned}$$

Bonus. (5pts) An old woman in Jakarta says: "It will rain today with 80% chance. If it rains today, it will rain tomorrow with 70% chance. If it doesn't rain today, then tomorrow's rain will come with 90% chance." What is the probability that it rains on exactly one of the days?

$$\begin{aligned} &P\left(\left(\begin{array}{c} \text{rain} \\ \text{today} \end{array} \text{ and } \begin{array}{c} \text{no rain} \\ \text{tomorrow} \end{array}\right) \text{ or } \left(\begin{array}{c} \text{no rain} \\ \text{today} \end{array} \text{ and } \begin{array}{c} \text{rain} \\ \text{tomorrow} \end{array}\right)\right) \\ &= P\left(\begin{array}{c} \text{rain} \\ \text{today} \end{array} \text{ and } \begin{array}{c} \text{no rain} \\ \text{tomorrow} \end{array}\right) + P\left(\begin{array}{c} \text{no rain} \\ \text{today} \end{array} \text{ and } \begin{array}{c} \text{rain} \\ \text{tomorrow} \end{array}\right) \\ &= P(\text{rain today}) \cdot P(\text{no rain tomorrow} \mid \text{rain today}) + P(\text{no rain today}) \cdot P(\text{rain tomorrow} \mid \text{no rain today}) \\ &= 0.8 \cdot 0.3 + 0.2 \cdot 0.9 \\ &= 0.24 + 0.18 = 0.42 \end{aligned}$$