Spring '05/MAT 117/Exam 2b **Name:** Solution *Show all your work.*

$F = P(1+rt)$ $F = P\left(1+\frac{r}{n}\right)^{nt}$ $F = D\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$ $P = R\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$ $APY = \left(1+\frac{r}{n}\right)^{n}-1$

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1. (4pts) Solve the equation, rounding the answer to 6 significant digits.

$1.05^{4t} = 2.25$ $\quad | \log$

$\log 1.05^{4t} = \log 2.25$

$4t \log 1.05 = \log 2.25$

$t = \dfrac{\log 2.25}{4 \log 1.05} = 4.15519$

2. (4pts) What is the future value of \$500 deposited for 15 months in an account bearing simple interest of 10%?

$F = 500 \cdot \left(1 + 0.10 \cdot \frac{15}{12}\right) = 562.50$

3. (5pts) A woman deposits \$4,000 into an account bearing a simple interest rate of 8%. How long will it take until she has \$5,000 in the account?

$5000 = 4000(1 + 0.08t) \quad | \div 4000$

$1.25 = 1 + 0.08t \quad | -1$

$0.25 = 0.08t \quad | \div 0.08$

$\frac{0.25}{0.08} = t$

$t = 3.125$ years

4. (5pts) What is the annual percentage yield of an account bearing 3.83% interest if it is compounded weekly?

$$APY = \left(1 + \frac{0.0383}{52}\right)^{52} - 1 = 0.039021\ldots$$

$$APY = 3.90\%$$

5. (6pts) Peter would like to save \$18,000 to buy a new car. He can get a savings account bearing 6% compounded quarterly. How much should he deposit at the end of every quarter in order to have \$18,000 after 3 years?

$$18000 = D\ \frac{\left(1 + \frac{0.06}{4}\right)^{4\cdot 3} - 1}{\frac{0.06}{4}} \quad \text{(systematic savings)}$$

$$18000 = D \cdot \frac{1.015^{12} - 1}{0.015}$$

$$18000 = D \cdot 13.04\ldots \quad \div 13.04\ldots$$

$$\frac{18000}{13.04\ldots} = D$$

$$D = 1380.24$$

6. (6pts) If inflation averages 3% over the next 5 years, how much will an item costing \$10 today cost in 2010?

$$F = 10 \cdot \left(1 + \frac{0.03}{1}\right)^{1\cdot 5} = 10(1.03)^5 = 11.59$$

7. (12pts) The bored teenagers from the last exam decided to go and fight a bull one night. Sneaking around a cattle farm in the darkness, they accidentally rammed the van of an animals' rights group (with protesters sleeping inside), and caused \$13,000 worth of damage and medical expenses. In order to cover the damage, they took out a 5-year loan at 9% interest, compounded monthly.

a) What is their monthly payment?

b) What is the balance on the loan after 3 years?

c) Use your result from a) to figure out what their monthly payment would be, had the damage been only \$6,500.

a) Loan formula

$$13{,}000 = R \cdot \frac{1-\left(1+\frac{0.09}{12}\right)^{-12\cdot 5}}{\frac{0.09}{12}}$$

$$13000 = R\,\frac{1-(1.0075)^{-60}}{0.0075}$$

$$13000 = R \cdot 48.1733\ldots \quad \left| \div 48.1733\ldots \right.$$

$$\frac{13{,}000}{48.17\ldots} = R$$

$$R = 269.86$$

b) Balance = present value of remaining payments (2 years' worth)

$$P = 269.86\,\frac{1-(1.0075)^{-12\cdot 2}}{0.0075}$$

$$= 269.86 \cdot 21.889\ldots$$

$$= 5907.00$$

c) Payment for a $\frac{1}{2}$ loan amount

$= \frac{1}{2}$ Payment

$= \frac{1}{2} \cdot 269.86 = 134.93$

8. (8pts) What annual interest rate should a deposit be earning in order for it to triple in 8 years? Assume the compounding is done semiannually.

$3 = 1 \cdot \left(1 + \frac{r}{2}\right)^{2 \cdot 8}$ (compound interest formula)

$3 = \left(1 + \frac{r}{2}\right)^{16} \quad | \ (\ )^{\frac{1}{16}}$

$3^{\frac{1}{16}} = 1 + \frac{r}{2} \quad | -1$

$0.07107\ldots = \frac{r}{2} \quad | \cdot 2$

$0.14215\ldots = r$

$r = 14.22\%$

Bonus. (5pts) A couple of newlyweds took out a 20-year, \$154,000 loan to finance their new home. The interest rate on this loan is 5.75% compounded monthly, making their monthly payment \$1081.21. Write the amortization schedule for the first three payments.

Payment	Amount	Toward interest	Toward Principal	Balance
1	1081.21	737.92	343.29	153,656.71
2	1081.21	736.27	344.94	153,311.77
3	1081.21	734.62	346.59	152,965.18