

1. (7pts) A professor is considering whether to expand, decrease or maintain the current amount of homework for a particular course. A survey of students resulted in the following table showing the options they approved of:

Percent of voters:	8	39	17	34	2
Expand	X				X
Decrease		X	X		X
Maintain	X	X		X	X

- a) Which option wins using the approval method?
 b) What percentage of voters has no influence on the outcome?

a) E $8+2=10$

D $39+17+2=58$

M $8+39+34+2=83$

b) The 2% that approved of all choices

2. (6pts) Suppose three candidates are running in an election decided by plurality with a runoff between the two top finishers. If the results of the first ballot are Godfrey 214, Smith 140, Mawson 185, what percentage of Smith supporters need to vote for Godfrey in order for Godfrey to win the election?

G 214

S 140

M 185

Even if $214-185=29$ of Smith's

supporters vote for Godfrey,

Godfrey will win if more than half of remaining 111 vote for them.

$$\frac{111}{2} = 55.5$$

G. needs 56 votes to win

$$\frac{56}{140} = 0.4 \quad \text{or } 40\%$$

3. (12pts) A gourmet cooks' association is trying to elect the recipient of their "Chef of the year" prize. Their preference rankings for the four candidates are as follows:

Number of votes:	5	2	5	4	3
Byte	1	1	4	3	4
Chow	2	3	1	4	2
Suallo	4	4	2	1	3
Nosch	3	2	3	2	1

- a) Who wins using the plurality method?
 b) Who wins using the plurality method, followed by a runoff of the two top finishers?
 c) Can the ~~four~~^{three} cooks who ranked Nosch first obtain a preferable outcome if they voted strategically, assuming all the other cooks voted as shown in the table?

a) B 7 ← wins
 C 5
 S 4
 N 3

b) Ranking Chow first will get Chow in the runoff, but will then lose like in b).

Ranking Suallo first gives the score:

B 7 → B 7
 C 5 to runoff
 S 7 → S 7+5=12
 N 0

Suallo would win, which is a preferable outcome to Byte.

4. (5pts) If 53 votes are cast, what is the smallest number of votes a winning candidate can have in a three-candidate race that is decided by plurality? Justify your answer.

Winning by the smallest no. of votes occurs when all candidates have an approximately equal number of votes. Thus:

$$\frac{53}{3} = 17, \text{ rem } 2$$

The remaining 2 votes have to go to one candidate to prevent a tie, hence, 19 votes are needed to win.

5. (15pts) A group of bored teenagers are deciding on what to use as a dare.¹ The choices are "jump off a bridge", "drink kerosene" and "fight a bull". Their preference rankings are shown below.

	Percentage of votes:	15	3	16	28	31	7
3	Jump off bridge	1	1	2	3	2	3
1	Drink kerosene	2	3	1	1	3	2
2	Fight bull	3	2	3	2	1	1

- Which dare is the Condorcet winner, if any?
- Which dare wins using the Borda method?
- Perform the check on the sum of Borda points.
- Can the 3% of teens who ranked "jump off a bridge" first and "fight a bull" second obtain a preferable outcome using the Borda method if they voted strategically, assuming all the other players voted as shown in the table?
- For a fun poll, mark your own rankings on the left of the table.

$$\begin{aligned}
 a) \quad & \left. \begin{array}{l} J \quad 18 + 31 = 49 \\ D \quad 44 + 7 = 51 \text{ w} \end{array} \right\} \\
 & \left. \begin{array}{l} J \quad 18 + 16 = 34 \\ F \quad 38 + 28 = 66 \text{ w} \end{array} \right\} \\
 & \left. \begin{array}{l} D \quad 44 + 15 = 59 \text{ w} \\ F \quad 38 + 3 = 41 \end{array} \right\}
 \end{aligned}$$

$$b) \quad J \quad 3 \cdot 18 + 2 \cdot 47 + 1 \cdot 35 = 183$$

$$D \quad 3 \cdot 44 + 2 \cdot 22 + 1 \cdot 34 = 210 \text{ wins}$$

$$F \quad 3 \cdot 38 + 2 \cdot 31 + 1 \cdot 31 = 207$$

600 \swarrow equal

$$c) \quad (100 \text{ "voters"}) \cdot (6 \text{ pts per vote}) = 600$$

"Drink kerosene"
is the Condorcet
winner.

d) They could rank "Fight bull" first

	Original score	3% voters' contribution	w/o 3% voters	if they change vote	new totals
J	183	-9	174	+6	180
D	210	-3	207	+3	210
F	207	-6	201	+9	210

At best, they can get a tie.

¹Fictional morons. Do not attempt.

if they rank

J	2
D	3
F	1

6. (5pts) Suppose there are 50 votes cast in an election between three candidates, decided by plurality. After the first 40 votes are counted, the tally is Heath 17, Chang 14 and Packer 9. What is the minimal number of remaining votes Chang needs to be assured of a win? Justify your answer.

H 17

C 14

P 9

Chang needs 3 votes to catch up, then,
of the remaining 7, they need 4 to win.

Thus, Chang needs $3+4=7$ votes to win.

Bonus. (5pts) Devise a scenario with three candidates that shows that plurality does not satisfy the property of independence from irrelevant alternatives. (Recall that the property says: if A wins over B in a two-candidate race, then in a race with any additional candidates, B cannot win.) Your answer should be a table with preference rankings for the three candidates, along with a tally of votes showing the property above is violated.

Ex: A 2

B 3

C 2, prefer A to B

	2	3	2
A	1	2	2
B	2	1	3
C	3	3	1

With just A, B:

A $2+2=4$ wins

B 3

A 2

B 3 wins

C 2