1. (7pts) A professor is considering whether to expand, decrease or maintain the current amount of homework for a particular course. A survey of students resulted in the following table showing the options they approved of:

Percent of voters:	8	39	17	34	2
Expand	X				X
Decrease		X	X		X
Maintain	X	X		X	X

a) Which option wins using the approval method?
b) What percentage of voters has no influence on the outcome?

a) E $8+2=10$
   D $39+17+2=58$
   M $8+39+34+2=83$ wins

b) The 2% that
   voted for all options.

2. (6pts) Suppose three candidates are running in an election decided by plurality with a runoff between the two top finishers. If the results of the first ballot are Godfrey 214, Smith 140, Mawson 185, what percentage of Smith supporters need to vote for Mawson in order for Mawson to win the election?

G 214
S 140
M 185

Mawson needs $214-185=29$ votes to catch up

Of the remaining 111 Smith votes, M. needs 56 to win.

Thus $56+29=85$, $\frac{85}{140}=0.6071$, so 60.71%

3. (12pts) A gourmet cooks' association is trying to elect the recipient of their "Chef of the year" prize. Their preference rankings for the four candidates are as follows:

Number of votes:	5	2	5	4	3
Byte	1	1	4	3	4
Chow	2	3	1	4	2
Suallo	4	4	2	1	3
Nosch	3	2	3	2	1

a) Who wins using the plurality method?
b) Who wins using the plurality method, followed by a runoff of the two top finishers?
c) Can the five cooks who ranked Chow first obtain a preferable outcome if they voted strategically, assuming all the other cooks voted as shown in the table?

a) B 7 wins ⟶ b) B 7+4 = 11 ⟵ wins
   C 5 ⟶   C 5+3 = 8
   S 4   runoff
   N 3

c) If they rank   B 4   plurality gives   B 7 ⟶ runoff ⟶ B 7
                  C 2   C 0
                  S 1   S 9 ⟶ S 9+3 = 12
                  N 3   N 3

Their 2nd choice would win!

4. (5pts) If 53 votes are cast, what is the smallest number of votes a winning candidate can have in a three-candidate race that is decided by plurality? Justify your answer.

One wins with the smallest number of votes if they are evenly distributed among candidates.

$\frac{53}{3} = 17$, rem. 2

Winning candidate needs 17+2 = 19 votes (to avoid a tie)

5. (15pts) A group of bored teenagers are deciding on what to use as a dare.[1] The choices are "jump off a bridge", "drink kerosene" and "fight a bull". Their preference rankings are shown below.

Percentage of votes:	15	3	16	28	31	7
3 Jump off bridge	1	1	2	3	2	3
1 Drink kerosene	2	3	1	1	3	2
2 Fight bull	3	2	3	2	1	1

a) Which dare is the Condercet winner, if any?
b) Which dare wins using the Borda method?
c) Perform the check on the sum of Borda points.
d) Can the 31% of teens who ranked "fight a bull" first and "jump off a bridge" second obtain a preferable outcome using the Borda method if they voted strategically, assuming all the other players voted as shown in the table?
e) For a fun poll, mark your own rankings on the left of the table.

a) J $18 + 31 = 49$
   D $44 + 7 = 51$ w

   J $18 + 16 = 34$
   F $38 + 28 = 66$ w

   D $44 + 15 = 59$ w
   F $38 + 3 = 41$

   Drink Kerosene
   is the Condercet
   winner.

b) J $3 \cdot 18 + 2 \cdot 47 + 1 \cdot 35 = 183$
   D $3 \cdot 44 + 2 \cdot 22 + 1 \cdot 34 = 210$ wins
   F $3 \cdot 38 + 2 \cdot 31 + 1 \cdot 31 = \dfrac{207}{600}$

c) $(100 \text{"voters"}) \cdot (6 \text{ points per ballot}) = 600$

d)

	31 voters contribute	w/o 31 voters	altered vote	new total
J 183	-62	121	$+93$	214
D 210	-31	179	$+31$	210
F 207	-93	114	$+62$	-176

↑

Suppose   J 1
they vote D 3
           F 2

They could get J to win

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[1] Fictional morons. Do not attempt.

6. (5pts) Suppose there are 50 votes cast in an election between three candidates, decided by plurality. After the first 40 votes are counted, the tally is Heath 17, Chang 14 and Packer 9. What is the minimal number of remaining votes Heath needs to be assured of a win? Justify your answer.

In the worst-case scenario, Chang gets 3 votes and catches up.

Heath needs 4 of the remaining 7 votes to win.

Thus, Heath needs 4 votes.

Bonus. (5pts) Devise a scenario with three candidates that shows that plurality with runoff does not satisfy the property of independence from irrelevant alternatives. (Recall that the property says: if A wins over B in a two-candidate race, then in a race with any additional candidates, B cannot win.) Your answer should be a table with preference rankings for the three candidates, along with a tally of votes showing the property above is violated.

Under scenario, presence of C should cause a runoff between B and C. With only A and B present, A should win.

A 2, prefs B to C

B 4

C 3 prefs A to B

	2	4	3
A	1	2	2
B	2	1	3
C	3	3	1

A 5   } head-to-head
B 4

A 2
B 4 → B 4+2=6 wins
C 3 → C 3
   runoff