

# The 54th Spring Topology and Dynamics Conference

March 18–21, 2020  
Murray State University  
Murray, KY, USA

March 5, 2020

## Abstracts

### Plenary talk/Semi-Plenary talk/ Topology in Data Science Workshop

**Hussam Abobaker**

Virginia Tech University

A Survey of the Set Function  $T$

We give a survey of the set function  $T$  and present some open problems.

**Henry Adams**

Colorado State University

Topology in Data Science Workshop

Part one of this workshop will be an introduction to applied topology. I will introduce Čech (nerve) simplicial complexes, Vietoris-Rips (clique) simplicial complexes, persistent homology, the stability theorem, and how these techniques can be used to approximate the shape of a dataset when given only a finite sample. A motivating example is the conformation space of the cyclo-octane molecule, which is a Klein bottle glued to a 2-sphere along two circles of singularity.

In part two of the workshop, I will advertise open questions in applied topology for which tools from geometric group theory, shape theory, quantitative topology, and equivariant topology are relevant. If a point cloud is sampled from a manifold, then as more samples are drawn, the persistent homology of the Vietoris-Rips complex of the point cloud converges to the persistent homology of the Vietoris-Rips complex of the manifold. But what are the homotopy types of Vietoris-Rips complexes of manifolds? Essentially nothing is known, except that as the scale parameter increases, the Vietoris-Rips complex of the circle obtains the homotopy types of the circle, the 3-sphere, the 5-sphere, the 7-sphere, ..., until finally it is contractible. I will connect Vietoris-Rips complexes of manifolds to the filling radius, which Gromov used to prove the systolic inequality in 1983, and to Borsuk-Ulam theorems for maps from spheres into higher-dimensional codomains.

**Ana Anušić**

University of São Paulo, Brazil

Local structure of inverse limit spaces

I will give an overview of what we know about local structure of inverse limits  $\varprojlim(G, f)$ , where  $G$  is a finite graph, and  $f: G \rightarrow G$  is a continuous function. Such spaces can be realized as global

attractors of  $\mathbb{R}^3$  (sometimes  $\mathbb{R}^2$ ) homeomorphisms. The main goal is to characterize existence, number, and type of points which are non-solenoidal, i.e. not locally homeomorphic to a zero dimensional set of arcs. For example, such points are often (but not always) limit points of the iterates of the critical set of  $f$ , while the recurrence of the critical set will often indicate the existence of endpoints. There are still a lot of open questions which I will discuss. Most of the results so far were obtained for a very special case of unimodal interval inverse limit spaces, and even there the situation is not completely clear. We had some recent progress in that direction (joint work with Lori Alvin, Henk Bruin and Jernej Činč), and have also obtained some generalizations to finitely piecewise monotone interval inverse limits (with Jernej Činč).

**Aleksandr Berdnikov**

Parsimonious cones

Massachusetts Institute of Technology

For any simplicial complex  $X^n$  with bounded degrees there is a triangulated (with bounded degrees) ball  $B^{2n+2}$  that contains  $X$  on its boundary as a subcomplex, and employs almost as many simplexes as  $X$  does. This construction can be used to construct compact embeddings of complexes, and thus build peculiar triangulations of spaces, and provides an efficient method to deform complexes to force their degrees to be bounded, i. e. locally simplify the triangulation.

**Lei Chen**

Actions of Homeo and Diffeo groups

California Institute of Technology

on manifolds

**Coauthors: Kathryn Mann**

In this talk, I discuss the general question of how to obstruct and construct group actions on manifolds. I will focus on large groups like Homeo(M) and Diff(M) about how they can act on another manifold N. The main result is an orbit classification theorem, which fully classifies possible orbits. I will also talk about some low dimensional applications and open questions.

**Steven Clontz**

Limited information strategies

University of South Alabama

for infinite-length games

Several ideas from topology and set theory may be characterized by considering two-player infinite-length games. During each round  $n \in \omega = \{0, 1, 2, \dots\}$ , suppose Player 1 makes a move  $a_n$  (perhaps choosing an open cover of a given regular space), followed by Player 2 making a move  $b_n$  (perhaps choosing a finite subcollection from 1's chosen cover); the winner of such a game is determined by the sequence of moves  $a_0, b_0, a_1, b_1, \dots$  (perhaps Player 2 wins if their choices form a cover).

The topological game specified above is known as Menger's game, and Player 2 has an unbeatable strategy that only uses information limited to the round number and the most recent move of Player 1 in this game if and only if the given regular space is  $\sigma$ -compact. In this talk, we will explore various results of this flavor found in the literature, including an interesting game-theoretic proof appropriate for undergraduates that the real numbers are uncountable.

**Spencer Dowdall**

Dynamical invariants via universal trees

Vanderbilt University

for free-by-cyclic groups

**Coauthors: Ilya Kapovich and Christopher Leininger**

Fully irreducible free group automorphisms have many interesting dynamical invariants. These include a stretch factor that measures the exponential growth of word length under iteration, and canonical attracting/repelling fixed points in compactified Outer Space, that is, the parameter

space of  $\mathbb{R}$ -trees equipped with isometric actions of the free group. This talk will introduce these invariants and explore how they vary for different free-by-cyclic splittings of a fixed group  $G$ . For splittings in the same component of the BNS-invariant of  $G$ , we will see that the associated attracting  $\mathbb{R}$ -trees are all topologically equivalent and that this universal topological tree can be used to define a polynomial invariant that simultaneously encodes the associated stretch factors.

**Andrew Dykstra**

Symbolic Dynamical Systems of Linear Complexity

Hamilton College

**Coauthors: Nic Ormes and Ronnie Pavlov**

For a symbolic dynamical system, the word complexity function  $c(n)$  counts the number of allowed words of length  $n$ . In this talk we consider examples that are topologically transitive, and where  $c(n)$  grows linearly. We show how the growth rate of  $c(n)$  can be used to bound both the number of proper minimal subsystems the system can have, and the number of ergodic measures it can support. Our bounds generalize those of Boshernitzan and are closely related to those of Cyr and Kra.

**Vera Fischer**

Combinatorial sets of reals and their spectrum

Kurt Gödel Research Center

Maximal almost disjoint families, maximal cofinitary groups and maximal independent families are among the combinatorial sets of reals, which are central to the study of the combinatorial properties of the real line. In this talk, we will discuss recent developments regarding the possible cardinalities of such extremal sets of reals, as well as their definability properties.

**Olga Kharlampovich**

Automorphisms of free group

CUNY Hunter College and Graduate Center

and first-order properties  
of tuples of elements

**Coauthors: C. Natoli**

We study to which extent the first-order properties of an  $n$ -tuple  $\bar{a}$  in a non-abelian free group determine its automorphic orbit. By the results of Perin-Sklinos and Ould Houcine, the free group is homogeneous, namely the first-order type of a tuple determines this tuple up to automorphism. We prove that non-abelian free groups of finite rank at least 3 or of countable rank are not  $\forall$ -homogeneous (free group of rank 2 is  $\forall$ -homogeneous by the result of Nies). We also provide interesting examples of countable non-finitely generated groups elementary equivalent to free groups.

**Christopher Leininger**

Polygonal billiards,

University of Illinois at Urbana-Champaign

Liouville currents, and rigidity

**Coauthors: Moon Duchin, Viveka Erlandsson, and Chandrika Sadanand**

A particle traveling inside a Euclidean polygon gives rise to a biinfinite “bounce sequence” (or “cutting sequence”) recording the (labeled) sides encountered by the particle. In this talk, I will describe work with Duchin, Erlandsson, and Sadanand, in which we prove that the set of all bounce sequences—the “bounce spectrum”—essentially determines the shape of the polygon. This is consequence of our main result about Liouville currents on surfaces associated to nonpositively curved Euclidean cone metrics. In the talk I will explain the objects mentioned above, how they relate to each other, and give some idea of what goes into the proofs.

**Piotr Minc**  
Auburn University

A planar continuum with a simple dense canal  
under each embedding in the plane

**Coauthors: Jernej Činč**

It is still unknown whether each non-separating plane continuum has the fixed point property. The strongest partial result toward solving the problem was proven independently by H. Bell (1967), K. Sieklucki (1968) and S. Iliadis (1970). They proved that every non-separating plane continuum without the fixed point property has a simple dense canal. In 1983, B. Brechner and J. C. Mayer asked whether there exist a non-separating planar continuum such that every embedding of it in the plane has a simple dense canal. During the talk we will outline a construction of such an example.

So, what does it mean that a plane continuum  $X$  has a dense simple canal  $C$ ? Imagine that  $X$  is an island and  $C$  is a fjord that cuts into  $X$  becoming narrower and narrower as you paddle your kayak along it. It lets you go forward, and never turn back or circle. Since it is dense, you will eventually be arbitrarily close to each point on the island.

**Ronnie Pavlov**  
University of Denver

Entropy and factors of subshifts on amenable groups

**Coauthors: Uijin Jung and Kevin McGoff**

I will present some recent work on entropies of factors of dynamical systems on amenable groups. For every countable amenable group  $G$ , we show that every  $G$ -subshift  $X$  has subshift factors achieving a set of entropies dense in the interval  $[0, h(X)]$ , and that any zero-dimensional  $G$ -system  $X$  has zero-dimensional factors achieving every entropy in the interval  $[0, h(X)]$ .

We will give definitions of all relevant concepts, and also provide some context about how these results fit into the overall body of work on the so-called ‘lowerability of entropy’ question, first examined by Shub and Weiss.

**D. Columba Pérez Flores**  
Instituto de Matemáticas, UNAM

On Countable Dynamical Systems

**Coauthors: Chris Good**

$(X, f)$  is called a *countable dynamical system* if  $X$  is a countable compact Hausdorff space and  $f : X \rightarrow X$  a continuous function. Countable dynamical systems appear naturally in different situations, e.g. as subsystems of onto interval maps. Interestingly, most of the dynamical properties of this type of systems rely on the scattered nature of their phase space. In this talk we will look at concepts such as expansivity, transitivity and shadowing in the context of countable dynamical systems, mentioning both old and new results on them.

**Iian Smythe**  
Rutgers University

Equivalence of generic reals

We will discuss how the method of forcing, originally developed to prove independence results in set theory, can be used to construct a family of countable Borel equivalence relations on “generic” sets of reals, each corresponding to a different notion of forcing. In the case of Cohen forcing, which involves a topological notion of genericity, we will show that the resulting equivalence relation is of relatively low complexity within the hierarchy of countable Borel equivalence relations. In the case of random forcing, where the notion of genericity is measure-theoretic, the resulting equivalence relation is comparatively complex. These results and the methods used to prove them are closely related to well-known open questions concerning the complexity of unions of hyperfinite equivalence relations and of Turing equivalence.

**Jing Tao**

University of Oklahoma

**Coauthors: Camille Horbez**

In the 1970s, Thurston generalized the classical classification of self-maps of the torus to surfaces of higher genus. This is known as the Thurston classification of surface homeomorphisms. Since Thurston's work, many alternative proofs have been given. Perhaps the most famous is due to Bers, who rephrased the classification in terms of an extremal problem on complex structures on surfaces. In joint work with Camille Horbez, we revisit Bers' approach but from the point of view of hyperbolic geometry. This gives a new proof of the Thurston classification as well as new structural results on pseudo-Anosov homeomorphisms.

Classification of surface homeomorphisms

## Continuum Theory

**Jernej Činč**

AGH Krakow and University of Ostrava

**Coauthors: Jan P. Boroński, Magdalena Foryś-Krawiec**

In this talk I will discuss continua which have rigid homeomorphism groups and admit minimal self-maps (= a self-map is called minimal if there are no non-empty proper closed subsets invariant under this map). Among other results I will describe a continuum that admits a minimal noninvertible map but has degenerate group of self-homeomorphisms.

On minimal rigid continua

**Tavish Dunn**

Baylor University

**Coauthors: David Ryden**

We examine versions of the Intermediate Value Property applicable to upper semicontinuous set-valued functions  $f : [0, 1] \rightarrow 2^{[0,1]}$ , providing sufficient conditions such that the corresponding inverse limit spaces are connected. Additionally we examine the relationship between the existence of indecomposable subcontinua of these inverse limits with a single bonding map with an Intermediate Value Property and the periodic cycles of the bonding map.

Inverse limits involving set-valued functions  
with an intermediate value property

**Sina Greenwood**

University of Auckland

**Coauthors: Michael Lockyer**

We investigate necessary and sufficient conditions for the inverse limit of set-valued functions to be path-connected. Some of these conditions involve mountain climbing and the notion of a path-component base which we will define. We give a characterisation of path-connected Mahavier products.

Path-connected inverse limits  
of set-valued functions on intervals

**Rodrigo Hernández-Gutiérrez**

Universidad Autónoma Metropolitana, Iztapalapa

**Coauthors: Logan Hoehn**

Recently, Logan Hoehn and Yaziel Pacheco-Juarez have given characterizations of some dendroids in terms of their homogeneity degree. We will restrict our attention to the class of smooth fans. On the opposite side of the homogeneity spectrum lie the so-called rigid spaces. While it can be easily proved that a smooth fan is never rigid, it might be interesting to investigate whether there

Almost rigid smooth fans

are smooth fans with properties close to rigidity. We define a smooth fan  $X$  to be almost rigid if every homeomorphism  $h : X \rightarrow X$  is the identity when restricted to the set of endpoints  $E(X)$ . In this talk we discuss some examples of rigid fans.

**Logan C. Hoehn**  
Nipissing University

Shortest paths in plane domains

**Coauthors: L. G. Oversteegen and E. D. Tymchatyn**

I will describe our recent work on shortest paths in arbitrary plane domains. We introduce several equivalent formulations of the concept of a shortest path, and state precisely our result on existence and uniqueness of a shortest path associated to any homotopy class. I will also allude to some ideas and results from complex analysis which we appealed to in the proof.

**Alejandro Illanes**

Continua with unique cone

Universidad Nacional Autónoma de México

**Coauthors: Verónica Martínez de la Vega and Daria Michalik**

A continuum  $X$  is said to have unique cone provided that the following implication holds: if  $Y$  is a continuum and  $\text{cone}(X)$  is homeomorphic to  $\text{cone}(Y)$ , then  $X$  is homeomorphic to  $Y$ . In this talk we present some characterizations of continua having unique cone.

Particularly, we consider the following families of continua: (a) locally connected curves, (b) indecomposable continua such that their non-degenerate proper sub continua are areas, and (c) fans.

**Teja Kac**  
University of Maribor

Big and large continua in inverse limits  
of inverse systems over directed graphs

**Coauthors: Iztok Banič, Matevž Črepnjak,  
Peter Goričan, Matej Merhar, Uroš Milutinović**

In the theory of generalized inverse limits it is a well-known fact that the generalized inverse limits may not be connected even if all the factor spaces are closed intervals. However, it has been shown recently by Banič and Kennedy that such generalized inverse limits always contain large continua, if the bonding functions have connected and surjective graphs. We present recent results that generalize the notion of generalized inverse limits of inverse sequences of closed intervals with upper semicontinuous bonding functions to inverse limits of inverse systems over directed graphs and show that under certain conditions, such inverse limits also contain large continua.

**Boštjan Lemež**  
University of Ljubljana

3-cell as the inverse limit of a set-valued function  
indexed by the integers

We construct an upper semicontinuous function  $f : [0, 1] \rightarrow 2^{[0,1]}$  such that the inverse limit of the inverse sequence of closed unit intervals with  $f$  as the bonding function indexed by the integers is a 3-cell. R. P. Vernon presented an example of a function such that a 2-cell is obtained as inverse limit and stated a question whether exist a function such that the inverse limit indexed by the integers is an  $n$ -cell for  $n > 2$ . We will also generalize the function  $f$  and obtain an  $n$ -cell for any positive integer  $n$ .

**Wayne Lewis**  
Texas Tech University

100+ Years of Indecomposable Continua

Indecomposable continua were defined 100 years ago. Some brief history is given and then several

problems concerning indecomposable and hereditarily indecomposable continua are discussed.

**David Lipham**

Endpoints of the Lelek fan

Auburn University at Montgomery

**Coauthors: Jan J. Dijkstra**

Given the endpoint set  $E$  and vertex  $v$  of any Lelek fan, we say that a topological space  $X$  is an *exploded endpoint space* if  $X$  is homeomorphic to a set  $X' \subseteq E$  such that  $X' \cup \{v\}$  is connected (in which case  $v$  is the *explosion point* for  $X' \cup \{v\}$ ). In this talk we will present new topological characterizations and features of these spaces. We will also discuss the topological types of several important examples, including Erdős spaces, homeomorphism groups of the Sierpinski carpet and Menger universal curve, and various escaping sets in complex dynamics.

**Marcus Marsh**

Weak chainability of arc folders

California State University, Sacramento

**Coauthors: C.L. Hagopian and J.R. Prajs**

A continuum is a compact, connected metric space. If a continuum  $X$  admits a monotone mapping  $\eta$  onto  $[0, 1]$ , we call the pair  $(X, \eta)$  a *continuum folder*. If for each  $t \in [0, 1]$ ,  $\eta^{-1}(t)$  is an arc or a point, we call  $(X, \eta)$  an *arc folder*. We give some introductory remarks concerning continuum folders, with particular attention to arc folders. We give some partial results to Rudy Gordh's 40 year-old question, "Do all arc folders have the fixed point property?" Then we show that arc folders are weakly chainable.

**Veronica Martínez de la Vega**

Topological Mixing and UPE

Instituto de Matemáticas, UNAM

**Coauthors: A. Illanes, D. Darji, J. Martínez-Montejano**

In this talk we study relationships between topological mixing and uniform positive entropy (UPE). For a compact metric space  $X$  with an open subset homeomorphic to the open interval  $(0, 1)$  and a mapping  $f : X \rightarrow X$  we show that the property of  $f$  weakly mixing and having the m-UPE property are equivalent. We show the same when  $X$  is a dendrite and  $f$  is an open mapping.

**Joe S. Ozbolt**

Planar Embeddings of the Knaster  $\Lambda V$ -Continuum

Auburn University

Planar embeddings of continua have been a common topic of interest in Continuum Theory since the beginning of the 20th century. Anušić, Bruin, and Činč have recently showed that any chainable continuum containing a nondegenerate indecomposable continuum can be embedded in the plane in uncountably many inequivalent ways. They asked which hereditarily decomposable chainable continua (HDCC) have uncountably many inequivalent planar embeddings. It was noted, as per the embedding technique of John C. Mayer with the  $\sin(1/x)$ -curve, that any HDCC which is the compactification of a ray with an arc has this property. We show here two methods for constructing  $\mathfrak{c}$ -many inequivalent planar embeddings of the classic Knaster  $\Lambda V$ -Continuum,  $K$ . The first of these two methods produces  $\mathfrak{c}$ -many planar embeddings of  $K$ , all of whose images have a different set of accessible points from the image of the standard embedding of  $K$ , while the second method produces  $\mathfrak{c}$ -many embeddings of  $K$  which preserve the set of accessible points of the standard embedding.

**Robert Roe**

Open Diameter Maps

Missouri University of Science and Technology

**Coauthors: Włodzimierz J. Charatonik, Ismail Uğur Tiriyaki**

Nadler, in *Hyperspaces of Sets*, asked to characterize continua that admit open diameter maps. We give an introduction to this problem, talk about our result that connected finite graphs admit open diameters maps and indicate directions for future work.

**David Ryden**  
Baylor University

Generalized Inverse limits  
and the Intermediate Value Property

Suppose  $f : [0, 1] \rightarrow 2^{[0,1]}$  is upper semicontinuous and surjective. The intermediate value property has been shown to restore to such functions some of the properties of classical continuous functions of  $[0, 1]$  onto itself. For example periodic cycles of such functions follows the Sarkovskii order, and the inverse limit of such functions is connected. In this talk, we show that such maps, when they are light, generate inverse limits with the full-projection property.

**Murat Tuncali**  
Nipissing University

Maps of rank  $\leq \mathfrak{m}$  revisited

**Coauthors: Pawel Krupski**

Let  $f : X \rightarrow Y$  be a function and let  $\mathfrak{m}$  be an infinite cardinal. Then we say that the rank  $r(f)$  of  $f$  is  $\leq \mathfrak{m}$  if

$$|\{y \in Y : |f^{-1}(y)| > 1\}| \leq \mathfrak{m}.$$

If  $\mathfrak{m} = \aleph_0$  then  $f$  is of countable rank. In this talk, we present some general properties and invariants of rank  $\leq \mathfrak{m}$  maps. We also show there are close relationships between them and monotone maps.

**Edward Tymchatyn**  
University of Saskatchewan

Inverse systems with simplicial bonding maps  
and cell structures

**Coauthors: Wojciech Debski, Kazuhiro Kawamura, Murat Tuncali**

For a topologically complete space  $X$  and a sufficiently large family of normal closed covers we give a construction of an inverse system of simplicial complexes with simplicial bonding maps such that the inverse limit is homotopy equivalent to  $X$ . A connection with cell structures (which are inverse systems of graphs ) is discussed.

**Scott Varagona**  
University of Montevallo

Inverse Limits with Smith Functions

We call an upper semi-continuous function  $f : [0, 1] \rightarrow 2^{[0,1]}$  a *Smith function* if  $f$  is surjective, the graph of  $f$  is connected, and the graph of  $f$  is the union of finitely many vertical and horizontal line segments. We investigate inverse limits whose bonding functions are Smith functions. Questions about connectedness, dimension, decomposability, etc., of such inverse limits will be considered. We also show how to construct indecomposable continua using inverse limits with Smith functions.

## Dynamical Systems

**Sourav Bhattacharya**  
University of Alabama at Birmingham

Very Badly Ordered Cycles of Interval Maps

**Coauthors: Alexander Blokh**

We prove that a periodic orbit  $P$  with coprime over-rotation pair is an over-twist periodic orbit iff



the  $P$ -linear map has the over-rotation interval with left endpoint equal to the over-rotation number of  $P$ . We show that this fails if the over-rotation pair of  $P$  is not coprime. We give examples of patterns with non-coprime over-rotation pairs, no block structure over over-twists, and with over-rotation number equal to the left endpoint of the forced over-rotation interval (call them *very badly ordered*, similar to patterns of degree one circle maps in [Alm98]). This presents a situation in which the results about over-rotation numbers on the interval and those about classical rotation numbers for circle degree one maps are different. In the end we explicitly describe the strongest unimodal pattern that forces a given over-rotation interval and use it to construct unimodal very badly ordered patterns with arbitrary non-coprime over-rotation pairs.

**Alexander Blokh**

University of Alabama at Birmingham

**Coauthors: Lex Oversteegen and Vladlen Timorin**

On cutpoints of subcontinua  
of polynomial Julia sets

We show that under some circumstances periodic cutpoints of subcontinua of connected Julia sets are cutpoints of the Julia sets themselves (e.g., this holds if subcontinua are polynomial-like Julia sets). For periodic points this extends old results by Levin-Przytycki originally obtained for periodic components of disconnected Julia sets.

**Javier Camargo**

Industrial University of Santander

**Coauthors: Johan Cancino**

The  $\omega$ -limit function on dendrites

Given a compact metric space  $X$  and a continuous function  $f: X \rightarrow X$ , we define the  $\omega$ -limit function  $\omega_f: X \rightarrow 2^X$  by  $\omega_f(x) = \omega(x, f)$  for each  $x \in X$ . We study the continuity of  $\omega_f$  when  $f$  is defined on a dendrite. Furthermore, we show some relationships between the connectedness of the set of periodic points and the equicontinuity of the function.

**Van Cyr**

Bucknell University

**Coauthors: Aimee Johnson, Bryna Kra, and Ayse Sahin**

Complexity thresholds for the emergence  
of dynamical properties in symbolic systems

The topological entropy of a subshift is the exponential growth rate of the number of words of different lengths in its language. For subshifts of entropy zero, finer growth invariants constrain their dynamical properties. In this talk we will examine how the complexity of a subshift affects its dynamical properties. In particular, we will see some recent results relating the word complexity of a subshift to its ability to carry measures lacking the Loosely Bernoulli property, as well as some applications.

**Jernej Činč**

AGH Krakow and University of Ostrava

**Coauthors: Ana Anušić, Henk Bruin**

Topological properties of Lorenz  
maps derived from unimodal maps

A symmetric Lorenz map is obtained by “flipping” one of the two branches of a symmetric unimodal map. We use this to derive a Sharkovsky-like theorem for symmetric Lorenz maps, and also to find cases where the unimodal map restricted to the critical omega-limit set is conjugate to a Sturmian shift. This has connections with properties of unimodal inverse limit spaces embedded as attractors of some planar homeomorphisms.

**Ian Grigsby**  
Baylor University

On Various Notions of Shadowing  
in Noncompact Spaces

**Coauthors: Jonathan Meddaugh**

A dynamical system is said to have the shadowing property if its approximate orbits are followed closely by true orbits. Various notions of this property arise depending on what we mean by an approximate orbit and on what sense of “closeness” we want a true orbit to have with respect to an approximate orbit. We discuss some notions of the shadowing property for dynamical systems on spaces that are not compact.

**Scott Kaschner**  
Butler University

Bifurcation Phenomena in Quadratic Rational Maps

This talk will survey known results regarding bifurcation in a family,  $\{f_{\lambda,t}\}_{t \in \mathbb{C}}$ , of quadratic rational maps with a fixed point multiplier of  $\lambda$ ; similar phenomena in another family will be presented. We will also discuss variety of attempts to describe resonance phenomena in these bifurcations.

**James Kelly**  
Christopher Newport University

Linear Operators with Infinite Entropy

**Coauthors: Will Brian**

We examine the chaotic behavior of certain continuous linear operators on infinite-dimensional Banach spaces, and provide several equivalent characterizations of when these operators have infinite topological entropy.

For example, it is shown that infinite topological entropy is equivalent to non-zero topological entropy for translation operators on weighted Lebesgue function spaces. In particular, finite non-zero entropy is impossible for this class of operators.

**Judy Kennedy**  
Lamar University

Characterizations of  $\mathcal{P}$ -like continua without the  
fixed point property in terms of open covers

**Coauthors: Iztok Banič, Uroš Milutinović, Piotr Minc**

We give two characterizations, in terms of open covers, of polyhedron-like continua without the fixed point property.

**Krystyna Kuperberg**  
Auburn University

Flows with bounded trajectories  
on non-compact 3-manifolds

Let  $\psi : M \rightarrow M$  be a flow on a non-compact 3-manifold  $M$ . A trajectory of  $\psi$  is bounded if its closure is compact.

In 1996, G. Kuperberg constructed a twisted plug in order to define flow compatible Dehn surgery. Using this method, he proved that every 3-manifold with empty boundary possesses a smooth, nonsingular, volume preserving flow with a discrete collection of circular trajectories.

The following generalizations of the above theorem will be presented:

*Every 3-manifold with empty boundary possesses a smooth, nonsingular, volume preserving flow with every trajectory bounded.*

**Daniel M. Look**  
St. Lawrence University

An Infinite Sequence of Wall-to-Wall  
Sierpiński Carpets for  $z \mapsto \lambda(z + \frac{1}{z})$

**Coauthors: None**

We prove that there is a sequence of purely imaginary parameter values converging to 0 such that the Julia set for  $z \mapsto \lambda(z + 1/z)$  is homeomorphic to the Sierpiński carpet fractal. Further, these Sierpiński carpet Julia sets are unbounded, giving us what we call Wall-to-Wall Sierpiński carpets.

**David McClendon**Speedups of  $\mathbb{Z}^d$ -odometers

Ferris State University

**Coauthors: Aimee S.A. Johnson**

Given a dynamical system  $(X, T)$ , a speedup of  $(X, T)$  is another dynamical system  $(X, T^p)$  where  $p : X \rightarrow \{1, 2, 3, \dots\}$ . We are interested two big-picture questions involving speedups: (1) to what degree must  $(X, T^p)$  be “equivalent” to  $(X, T)$ ? (2) given two dynamical systems, when does there exist a speedup of the first that is “equivalent” to the second? In this talk, we will briefly review results addressing both questions.

Then, we will discuss what is meant by a speedup of an action of  $\mathbb{Z}^d$ . In particular, we will give some results, related to the two questions listed above, for minimal  $\mathbb{Z}^d$ -Cantor systems, with a focus on speedups of  $\mathbb{Z}^d$ -odometers. One notable result generalizes work of Alvin, Ash and Ormes: we will show is that a bounded speedup of a  $\mathbb{Z}^d$ -odometer must be an odometer (as in the  $\mathbb{Z}$  case), but unlike the  $\mathbb{Z}$  case, the speedup need not be topologically conjugate to the original odometer.

**Jonathan Meddaugh**

Dynamics in shift spaces with countable alphabet

Baylor University

**Coauthors: Brian Raines**

In this talk we consider the dynamics of subshifts of Baire space, i.e. closed subsets of  $\mathbb{N}^\omega$  which are closed under the action of the shift map  $\sigma$ . These are natural generalizations of subshifts over finite alphabets and serve as a model for some non-compact dynamical systems.

In particular, we generalize a result of Lampart and Oprocha to this setting and demonstrate that in these systems, the weak specification property implies  $\omega$ -chaos.

**Lex Oversteegen**

Laminations and their critical portraits

University of Alabama at Birmingham

**Coauthors: Sasha Blokh and Vladlen Timorin**

A closed equivalence relation on the unit circle  $\mathbb{S}$  in the complex plane  $\mathbb{C}$  is said to be *laminal* if classes are finite and the convex hulls of distinct classes are disjoint. It is *d-invariant* if it is invariant under the covering map  $\sigma_d(z) = z^d$ . In that case the map  $\sigma_d$  induces a map  $f_d : \mathbb{C} \rightarrow \mathbb{C}$  on the quotient space, obtained by collapsing the convex hulls of classes to points, which we call a *topological polynomial*.

Thurston parametrized the space of all topological polynomials when  $d = 2$  by showing that each diameter, called a quadratic critical portrait, of the circle determines a unique topological polynomial. The space of all critical portraits is itself a circle and, if we identify two diameters who determine the same topological polynomial, the resulting quotient space of the circle parameterizes all quadratic topological polynomials. He showed that his space is a model for the space of quadratic polynomials acting on the complex plane. This model is conjecturally homeomorphic to the boundary of the Mandelbrot set.

As a first step to generalizing these results to the case  $d=3$  we study the relationships between two specific non-crossing chords in the unit disk, called a cubic critical portrait, and cubic topological

polynomials. The space of all cubic critical portraits is more complicated but well understood. We will show that in most cases a critical portrait also determines a unique cubic topological polynomial but there are countably many exceptions. We will also show that there are always at most finitely many topological polynomials determined by a given critical portrait and, if there is more than one, we show how they are related.

**Rachel L. Rossetti**

Quadratic rational maps that preserve an infinite measure

Agnes Scott College

**Coauthors: Jane Hawkins**

Generalized Boole transformations provide a complete characterization of all rational maps that preserve Lebesgue measure on the real line. Quadratic Boole functions admit no periodic orbits of period 2, so they are conformally conjugate to maps in the unique parameter space of rational maps with this property. We use this conformally conjugate form to provide a complete characterization of the  $c$ -isomorphism classes of quadratic Boole functions and pinpoint the role of Krengel entropy as a  $c$ -isomorphism invariant.

**Samuel Roth**

Dynamics on dendrites with

Silesian University in Opava

a closed set of endpoints

We construct dendrites with endpoint sets isometric to any totally disconnected compact metric space. This allows us to embed zero-dimensional dynamical systems into dendrites and solve a problem regarding Li-Yorke and distributional chaos.

**Sandeep Chowdary Vejandla**

Parameter space of

University of Alabama at Birmingham

Cubic Symmetric Laminations

**Coauthors: Alexander Blokh, Lex Oversteegen, Nikita Selinger and Vladlen Timorin**

To study the parameter space of all polynomials of degree  $d$  with connected Julia sets, Thurston proposed studying the space of all  $\sigma_d$ -invariant laminations where  $\sigma_d : S \rightarrow S$  is the degree  $d$  covering map of the unit circle defined by  $\sigma_d(z) = z^d$ . He completed this approach for the space of quadratic polynomials but the case of higher degree has remained elusive.

During this presentation, I will talk about the parameter space of cubic polynomials of the form  $z^3 + \lambda^2 z$  through **Cubic Symmetric Laminations**.

**Jim Wiseman**

Persistence of combinatorial Morse decompositions

Agnes Scott College

A Morse decomposition is a way to understand the relations among different subsets of the chain recurrent set of a continuous self-map on a metric space. A finite-resolution approximation of the map gives a combinatorial dynamical system, for which we can define a notion of Morse decomposition. We study the persistence properties of this decomposition as the grid resolution gets finer and finer, considering both the global structure and the dynamics on the individual elements.

**Kitty Yang**

Mapping class group of minimal subshifts

Northwestern University

**Coauthors: Scott Schmieding**

Let  $(X, \sigma)$  be a subshift. A flow equivalence of two spaces is an orientation-preserving homeomorphism of the suspension spaces. The mapping class group of a subshift is the group of self-flow

equivalences up to isotopy. We compute the mapping class group for various classes of minimal zero-entropy subshifts.

## Geometric Group Theory

**Ulysses Alvarez**

Binghamton University

**Coauthors: Ross Geoghegan**

The Up Topology on the Grassmannian Poset

The Grassmannian for  $k$  dimensional linear subsets of  $\mathbb{R}^n$  is  $G_k(\mathbb{R}^n)$ . Let  $\mathcal{P}$  denote the coproduct of  $G_k(\mathbb{R}^n)$ , where  $0 < k < n$ , considered as a topological poset partially ordered by inclusion. With the usual topology,  $\mathcal{P}$  has  $(n - 1)$  connected components. The *order complex*,  $\Delta(\mathcal{P})$ , is a connected complex; it is the subset of the join of  $G_k(\mathbb{R}^n)$  as  $k$  goes from 1 to  $n - 1$ , where  $z = \sum_{k=1}^{n-1} t_k z_k$  is in  $\Delta(\mathcal{P})$  if and only if  $i < j$  in  $\text{supp}(z)$  implies  $z_i < z_j$ . Combinatorialists are interested in a second (non-Hausdorff) topology on  $\mathcal{P}$ , called the *up* topology, which takes account of the poset structure, and makes  $\mathcal{P}$  connected. Theorem: Using the up topology on  $\mathcal{P}$ , there is a canonical weak homotopy equivalence  $f : \Delta(\mathcal{P}) \rightarrow \mathcal{P}$ . The theorem holds for more general posets (given the right hypotheses) and generalizes an old theorem of McCord for discrete posets. In this talk, I will sketch the proof of the theorem for the case when  $n = 3$  (i.e. lines and planes in  $\mathbb{R}^3$ ).

**Simon André**

Vanderbilt University

**Coauthors: Jonathan Fruchter**

Acylindrically hyperbolic groups  
and elementary equivalence

Zlil Sela proved that, given a non-cyclic torsion-free hyperbolic group  $G$ , the groups  $G$  and  $G * \mathbb{Z}$  are elementarily equivalent. In my talk, I will present the following partial generalization of this result: if  $G$  is acylindrically hyperbolic, say with no non-trivial normal finite subgroup, then  $G$  and  $G * \mathbb{Z}$  have the same  $\forall\exists$ -theory. It is an open question whether or not  $G$  and  $G * \mathbb{Z}$  have the same elementary theory.

**Lvzhou Chen**

University of Chicago

**Coauthors: Danny Calegari**

Dynamical methods in the study  
of big mapping class groups

Surfaces of infinite type, such as the plane minus a Cantor set, occur naturally in dynamics. However, their mapping class groups are much less studied and understood compared to the mapping class groups of surfaces of finite type. Some dynamical methods, such as Denjoy's construction and suspension tricks, are useful in the study of big mapping class groups. As a special case, for the mapping class group of the plane minus a Cantor set, we show that it contains all countable subgroups of  $\text{Homeo}^+(S^1)$ , and that it has a unique maximal nontrivial normal subgroup, which has uncountable index.

**Matthew B. Day**

University of Arkansas

**Coauthors: Andrew W. Sale and Richard D. Wade**

Calculating the virtual cohomological dimension of the  
automorphism group of a right-angled Artin group

We give an algorithm that takes in the defining graph of a RAAG and outputs the vcd of its automorphism group. The main tools are restriction maps (introduced by Charney–Crisp–Vogtmann),

and a short exact sequence for relative outer automorphism groups of RAAGs from previous work of Day–Wade. This work also involves the application of a theorem of Bieri on rational vcd, and a computation of the vcd of certain Fouxé-Rabinovitch groups using the Guirardel–Levitt outer space.

**Sami Douba**

2-Systems of arcs on spheres with prescribed endpoints

McGill University

Let  $S$  be an  $n$ -punctured sphere, with  $n \geq 3$ . We sketch a proof that  $\binom{n}{3}$  is the maximum size of a family of pairwise non-homotopic simple arcs on  $S$  joining a fixed pair of distinct punctures of  $S$  and pairwise intersecting at most twice. This is the dimension of an induced subcomplex of an “augmented” arc complex of  $S$ .

**Eduard Einstein**

Relatively Geometric Actions  
on CAT(0) Cube Complex

University of Illinois, Chicago

**Coauthors: Daniel Groves**

The recent study of hyperbolic groups acting on CAT(0) cube complexes has produced spectacular results such as Agol’s proof of the Virtual Haken Conjecture. A relatively hyperbolic group acts relatively geometrically on a CAT(0) cube complex if it admits a cocompact action on a CAT(0) cube complex with parabolic or finite cell stabilizers where parabolic subgroups act elliptically. Via Dehn filling techniques developed by Groves and Manning, groups that act relatively geometrically have a virtually special hyperbolic quotient that acts geometrically on a CAT(0) cube complex which can be used to help study the geometry of a relatively geometric action. We give a relatively geometric analogue of cubulation criteria developed by Bergeron and Wise that we use to show that finite volume hyperbolic 3-manifold groups act relatively geometrically on a CAT(0) cube complex. We provide an application of relatively geometric actions for characterizing finite-volume Kleinian groups. We will also discuss work in progress to prove relatively geometric analogues of structure theorems for cubulated hyperbolic groups developed by Wise and others. Finally, we conjecture that every relatively hyperbolic group with a geometric action on a CAT(0) cube complex admits a relatively geometric action on a CAT(0) cube complex.

**Elizabeth Field**

Trees, dendrites, and the  
Cannon-Thurston map

University of Illinois at Urbana-Champaign

When  $1 \rightarrow H \rightarrow G \rightarrow Q \rightarrow 1$  is a short exact sequence of three word-hyperbolic groups, Mahan Mitra (Mj) has shown that the inclusion map from  $H$  to  $G$  extends continuously to a map between the Gromov boundaries of  $H$  and  $G$ . This boundary map is known as the Cannon-Thurston map. In this context, Mitra associates to every point  $z$  in the Gromov boundary of  $Q$  an “ending lamination” on  $H$  which consists of pairs of distinct points in the boundary of  $H$ . We prove that for each such  $z$ , the quotient of the Gromov boundary of  $H$  by the equivalence relation generated by this ending lamination is a dendrite, that is, a tree-like topological space. This result generalizes the work of Kapovich-Lustig and Dowdall-Kapovich-Taylor, who prove that in the case where  $H$  is a free group and  $Q$  is a convex cocompact purely atoroidal subgroup of  $\text{Out}(F_n)$ , one can identify the resultant quotient space with a certain  $\mathbb{R}$ -tree in the boundary of Culler-Vogtmann’s Outer space.

**Meng-Che Ho**

Rationality of Growths of Groups

Purdue University

**Coauthors: Mark Pengitore, Seongjun Choi**

For a group  $G$  with a finite generating set  $S$ , we define  $b(n)$  to be the size of the  $n$ -ball in the Cayley graph of  $G$ . Gromov's polynomial growth theorem says that any group with  $b(n)$  bounded by a polynomial is virtually nilpotent. One then wonders when is  $b(n)$  precisely a polynomial, and this turns out to be related to the study of the rational growth of  $G$ . The group  $G$  is said to have rational growth if the power series associated with  $b(n)$  is a rational function. It was shown by Cannon that hyperbolic groups have rational growth in any generating set, and the same result was established by Benson for virtually abelian groups. However, Stoll gave a group which has rational growth in some, but not all, generating set. In this talk, we will discuss the rational growth of certain solvable groups, including the solvable Baumslag-Solitar groups and torus bundle groups.

**Jingyin Huang**

Ohio State University

**Coauthors: Bruce Kleiner, Stephan Stadler**

Higher rank Morse quasiflats

It is well-known that a triangle in a hyperbolic plane has the thin triangle property. Though the same property fails in a space  $X$  which is a product of two hyperbolic planes,  $X$  has the thin tetrahedron property, which is a higher dimensional generalization of thin triangle property. Such behavior of higher rank hyperbolicity was previously explored in the work of Kleiner and Lang, in the case of top rank quasiflats.

Here we go one step further and introduce the notion of higher rank Morse quasiflats, which unifies Morse quasigeodesics and top rank quasiflats studied by Kleiner and Lang. Simplest examples of Morse quasiflats are products of Morse quasigeodesics. Note that a typical Morse quasiflat is not necessarily Morse quasiconvex. Under appropriate assumptions on the ambient space we show that a number of alternative definitions are equivalent and quasi-isometry invariant; we also show that Morse quasiflats are asymptotically conical and have canonically defined Tits boundaries. We provide some first applications.

**Justin Lanier**

Georgia Tech

**Coauthors: Jim Belk, Dan Margalit, Becca Winarski**

Recognizing topological polynomials  
by lifting trees

A polynomial can be viewed as a branched cover of a sphere over itself that is compatible with a complex structure. If handed a topological branched cover of the sphere, we can ask whether it can arise from a polynomial — and if so, which one? We introduce a new topological approach that draws from the theory of mapping class groups of surfaces. By iterating a lifting map on a complex of trees, we are able to certify whether or not a given branched cover arises as a polynomial.

**Marissa Kawehi Loving**

Georgia Tech

**Coauthors: Shuchi Agrawal, Tarik Aougab,**

**Yassin Chandran, J. Robert Oakley, Roberta Shapiro, and Yang Xiao**

Automorphisms of the  $k$ -curve graph

The  $k$ -curve graph of an orientable surface  $S$  with negative Euler characteristic is a graph whose vertices correspond to (homotopy classes of) essential simple closed curves on  $S$ , and whose edges correspond to pairs of curves that geometrically intersect at most  $k$  times. We prove that the automorphism group of the  $k$ -curve graph of a surface  $S$  is isomorphic to the extended mapping class group for all  $k$  satisfying  $k \leq |\chi(S)| - 512$ .

**MurphyKate Montee**  
University of Chicago

Cubulating random groups at density  $d < 3/14$

For random groups in the Gromov density model at  $d < 3/14$ , we construct walls in the Cayley complex  $X$  which give rise to a non-trivial action by isometries on a CAT(0) cube complex. This extends results of Ollivier-Wise and Mackay-Przytycki at densities  $d < 1/5$  and  $d < 5/24$ , respectively. We are able to overcome one of the main combinatorial challenges remaining from the work of Mackay-Przytycki, and we give a construction that plausibly works at any density  $d < 1/4$ .

**Jean Pierre Mutanguha**  
University of Arkansas

Irreducible endomorphisms of  $F_n$  are hyperbolic

Mapping tori of irreducible free group automorphisms are well-understood. In this talk, I will sketch the reason why the mapping torus of an irreducible-nonsurjective free group endomorphism is word-hyperbolic. The proof uses various standard tools that are important in the study of low-dimensional dynamics and geometry: the Culler-Vogtmann outer space and its spine, laminations on graphs, and the Bestvina-Feighn combination theorem.

**Thomas Ng**  
Temple University

Constructing free semigroups in cube complexes

**Coauthors: Radhika Gupta and Kasia Jankiewicz**

Deciding when two given elements of a group generate a free group is generally very challenging even in hyperbolic 3-manifold groups. In light of the Tits alternative for cubical groups, all non-virtually abelian contain a free subgroup. In this talk, I will describe joint work with Radhika Gupta and Kasia Jankiewicz that characterizes the dynamics of pairs of isometries of cube complexes in order to construct free sub-semigroups. We will also discuss connections to effectivizing the dimension of cube complex on which a group can act as well as applications to uniform exponential growth of groups.

**Mark Pengitore**  
The Ohio State University

Residual Finiteness and Strict Distortion of  
Cyclic Groups in Solvable groups

We provide polynomial lower bounds for residual finiteness of residually finite, finitely generated solvable groups that admit infinite order elements in the Fitting subgroup of strict distortion at least exponential. For this class of solvable groups which include polycyclic groups with a nontrivial exponential radical and the metabelian Baumslag-Solitar groups, we improve the lower bounds found in the literature. Additionally, for the class of residually finite, finitely generated solvable groups of infinite Prufer rank that satisfy the conditions of our theorem, we provide the first nontrivial lower bounds.

**Jacob Russell**  
CUNY Graduate Center

The local-to-global property for Morse quasi-geodesics

**Coauthors: Davide Spriano and Hung C. Tran**

A hallmark of hyperbolic spaces is the nice behavior of their quasi-geodesic. In particular, both the Morse property (every quasi-geodesic is close to a geodesic) and the local-to-global property (every local quasi-geodesic is a global quasi-geodesic) characterize hyperbolic spaces. We examine a natural extension of these properties by studying the class of spaces in which every local Morse quasi-geodesic is a global Morse quasi-geodesic. We show that this class of spaces is incredibly rich, encompassing CAT(0) spaces, the mapping class group, Teichmüller space, and 3-manifold



groups. We also show a number of consequences for this local-to-global property for Morse geodesic, including a generalization of a theorem of Gitik to combinations of stable subgroups of  $CAT(0)$  groups, the mapping class group, and 3-manifold groups. In the case of the mapping class group, this produces a combination theorem for the topologically important convex cocompact subgroups.

**Chandrika Sadanand**

Univ. of Illinois Urbana Champaign

Rigidity of hyperbolic polygonal billiards and  
hyperbolic cone surfaces

**Coauthors: Moon Duchin, Viveka Erlandsson, Chris Leininger**

Consider a euclidean polygon-shaped billiard table on which a ball can roll along straight lines and reflect off of edges infinitely. In work joint with Moon Duchin, Viveka Erlandsson and Chris Leininger, we have characterized the relationship between the shape of a euclidean polygonal billiard table and the set of possible infinite edge-itineraries of balls travelling on it. This lead to a new rigidity result for flat cone metrics on surfaces. In this talk, we will explore the work-in-progress on the analogous questions for hyperbolic polygons, and hyperbolic cone metrics on surfaces.

**Jordan Sahattchiev**

**Coauthors: possibly Peter Scott**

Extensions of Stallings's Fibration Theorem

In this talk, I will hope to present some recent preliminary results of my work on proving a generalization of results contained in the Ph.D. theses of two of Peter Scott's former advisees: M. Moon and H. Elkalla. Their mathematical works are themselves non-trivial generalizations of John Stallings's celebrated Fibration Theorem. Recall that if a 3-manifold  $M$  is a surface bundle over the circle, we trivially have an onto homomorphism  $\pi_1(M) \rightarrow \mathbb{Z}$ , whose kernel is precisely the fundamental group of the fiber. In his 1961 paper, Stallings proved that under suitable hypotheses on  $M$ , the converse is also true. Namely, if the fundamental group of  $M$  admits a homomorphism onto  $\mathbb{Z}$ , then  $M$  fibers over the circle with fiber a surface whose fundamental group is commensurable with the kernel of this homomorphism. Loosely, Stallings proved his theorem under the assumption that  $M$  is a compact (note therefore that the fiber is, then, itself a compact surface). Moon and Elkalla proved generalizations under relaxed assumptions for the fundamental group of  $M$ . Compactness is generally needed in order to rule out pathological cases. This is still work in progress.

**Saldaña Sánchez**

Universidad Nacional Autónoma de México

Groups acting on trees and the  
Eilenberg-Ganea problem

Given a group  $G$ , it is a well-known theorem of Eilenberg and Ganea and a result of Swans establish that the cohomological dimension  $cd(G)$  and the geometric dimension  $gd(G)$  (the minimal dimension of a classifying space) are equal except for the possibility of having  $cd(G) = 2$  and  $gd(G) = 3$ . It is still not known whether there is a group satisfying  $cd(G) = 2$  and  $gd(G) = 3$ . If  $\mathcal{F}$  is a family of subgroups of  $G$ , we can define the cohomological and geometric dimension of  $G$  relative to  $\mathcal{F}$ , denoted  $cd_{\mathcal{F}}(G)$  and  $gd_{\mathcal{F}}(G)$  respectively. There is an Eilenberg-Ganea type theorem proved Lück and Meintrup in this relative context. If  $\mathcal{F}$  is the family of finite subgroups of the family of virtually cyclic subgroups, then Brady-Leary-Nucinkis and Fluch-Leary exhibited examples of groups satisfying  $cd_{\mathcal{F}}(G) = 2$  and  $gd_{\mathcal{F}}(G) = 3$ . In this talk we show how to construct more examples of groups with this property using as fundamental groups of graphs of groups (subject to certain restrictions), and Brady-Leary-Nucinkis groups as vertex groups.

**Takamichi Sato**

Waseda University

Isomorphism between the stabilizers of finite sets  
of numbers in Thompson's group  $F$

Since 1965 Thompson’s group  $F$  has been widely studied and has been shown to satisfy various exotic properties. G. Golan and M. Sapir studied subgroups of  $F$  which are the stabilizers of finite sets of real numbers in the interval  $(0, 1)$  under the natural action of  $F$  on  $(0, 1)$ . They described the algebraic structure of each stabilizer and also found a sufficient condition for the stabilizers to be isomorphic. In this talk we consider a necessary and sufficient condition for them to be isomorphic.

**Hung C. Tran**

University of Oklahoma

**Coauthors: Noel Brady**

We construct families of finitely presented groups exhibiting new divergence behavior; we obtain divergence functions of the form  $r^\alpha$  for a dense set of exponents  $\alpha \in [2, \infty)$  and  $r^n \log(r)$  for integers  $n \geq 2$ . The same construction also yields examples of finitely presented groups which contain Morse elements that are not contracting.

Divergence of finitely presented groups

**Yvon Verberne**

University of Toronto

Thurston obtained the first construction of pseudo-Anosov homeomorphisms by showing the product of two parabolic elements is pseudo-Anosov. A related construction by Penner involved constructing whole semigroups of pseudo-Anosov homeomorphisms by taking appropriate compositions of Dehn twists along certain families of curves. In this talk, I will present a new construction of pseudo-Anosov homeomorphisms and discuss some of the unique properties associated to maps obtained through this new construction.

Constructing pseudo-Anosov homeomorphisms  
using positive twists

**Derrick Wigglesworth**

University of Arkansas

**Coauthors: Edgar Bering and Yulan Qing**

Consider a punctured surface  $\Sigma$ , with (necessarily free) fundamental group  $F_n$ . A very basic connection between  $\text{MCG}(\Sigma)$  and  $\text{Out}(F_n)$  is provided by considering the action of a mapping class on the level of fundamental groups. I will discuss recent joint work with E. Bering and Y. Qing where we provide an algorithm to answer the following notably open question: Given an automorphism  $\phi$  of  $F_n$ , is  $\phi$  induced by a surface homeomorphism? Stated simply, “Is  $\phi$  geometric?”

Algorithmic Detection of Geometric Automorphisms of  $F_n$

## Geometric Topology

**Alexey Balitskiy**

Massachusetts Institute of Technology

**Coauthors: Aleksandr Berdnikov**

Celebrated waist estimates by Gromov and others assert that a map (between, say, Riemannian manifolds) dropping dimension must have a fiber of large volume. We discuss similar questions with volume of a fiber being replaced by its Urysohn  $d$ -width, a certain quantity measuring how effectively a space can be approximated by a  $d$ -dimensional one. For example, a map  $S^4 \rightarrow Y^1$  from the unit 4-sphere to a graph always has a fiber of large 1-width, but it might happen that all fibers have small 2-width and are topologically simple.

Waist estimates via the Urysohn width

**Neeraj Kumar Dhanwani**  
Indian Institute of Science Education  
and Research, Bhopal

Commuting conjugates  
of finite-order mapping classes

**Coauthors: Kashyap Rajeevsarathy**

Let  $\text{Mod}(S_g)$  be the mapping class group of the closed orientable surface  $S_g$  of genus  $g \geq 2$ . In this talk, we state necessary and sufficient conditions under which two finite-order mapping classes have representatives in their respective conjugacy classes that commute in  $\text{Mod}(S_g)$ . As an application, we show that any finite-order mapping class, whose corresponding orbifold is not a sphere, has a conjugate that is liftable under a finite cover. Furthermore, we show that any torsion element in the centralizer of an irreducible finite order mapping class is of order at most 2. We also state equivalent conditions for the primitivity of a torsion element of  $\text{Mod}(S_g)$ . Finally, we describe a procedure for determining the explicit hyperbolic structures that realize two-generator finite abelian groups of  $\text{Mod}(S_g)$  as isometry groups.

**Jerzy Dydak**  
University of Tennessee

Visual boundaries of geodesic spaces

I invented a unifying approach to all important compactifications. I will apply it to create a singular approach to boundaries of hyperbolic spaces and to boundaries of CAT(0)-spaces.

**Robin Elliott**  
Massachusetts Institute of Technology

The bar and cobar construction  
in Quantitative Topology

Quantitative Topology studies the relationship between topological invariants of metric spaces and the ‘metric complexity’ of these invariants. Some tools from topology, such as Sullivan’s rational homotopy theory, can readily be made quantitative. With other tools, the relationship between the topology and the metric is less clear. In this talk I will discuss how two such tools from topology apply in a quantitative setting: the bar and cobar constructions. Work of K.-T. Chen models the cohomology of loop spaces through the bar construction and I show that this gives quantitative bounds on the cohomology. Dually, work of Adams and Hilton models the homology of a loop space with the cobar construction. However here the quantitative implications are more subtle.

**Hanspeter Fischer**  
Ball State University

On the failure of the first Čech homotopy group to register  
geometrically relevant fundamental group elements

**Coauthors: Jeremy Brazas**

We construct a space  $\mathbb{P}$  for which the canonical homomorphism  $\pi_1(\mathbb{P}, p) \rightarrow \check{\pi}_1(\mathbb{P}, p)$  from the fundamental group to the first Čech homotopy group is not injective, although it has all of the following properties: (1)  $\mathbb{P} \setminus \{p\}$  is a 2-manifold with connected non-compact boundary; (2)  $\mathbb{P}$  is connected and locally path connected; (3)  $\mathbb{P}$  is strongly homotopically Hausdorff; (4)  $\mathbb{P}$  is homotopically path Hausdorff; (5)  $\mathbb{P}$  is 1-UV<sub>0</sub>; (6)  $\mathbb{P}$  admits a simply connected generalized covering space with monodromies between fibers that have discrete graphs; (7)  $\pi_1(\mathbb{P}, p)$  naturally injects into the inverse limit of finitely generated free monoids otherwise associated with the Hawaiian Earring; (8)  $\pi_1(\mathbb{P}, p)$  is locally free.

**Greg Friedman**  
Texas Christian University

What is sheaf cohomology?

This will be a very brief, but hopefully gentle, introduction to sheaf cohomology and some of the things it is good for. No prior experience with sheaves will be expected.

**Jonah Gaster**

Discrete harmonic maps

University of Wisconsin-Milwaukee

**Coauthors: Brice Loustau, Leonard Monsaingeon**

Combined work of Eels-Sampson and Hartman asserts the existence of a harmonic diffeomorphism in any homotopy class of maps between a pair of homeomorphic compact hyperbolic surfaces. I will discuss the background theory, present a suitable discretization, and locate discrete harmonic maps by applying a constant step gradient descent method, where convergence is guaranteed by explicit computations in the hyperbolic plane. In particular, we show that the discrete energy functional is strongly convex, a uniform statement not implied by the existing literature. Time permitting, I will discuss a computer implementation that exploits the above viewpoint, and the delicate problem of convergence of the discrete maps to the smooth harmonic map.

**Craig Guilbault**

End sums of open manifolds

University of Wisconsin-Milwaukee

**Coauthors: Jack Calcut and Patrick Haggerty**

Connected sums and boundary connected sums play important roles in the study of closed manifolds and manifolds with boundary, respectively. When working with open manifolds, a third variety of connected sum — the “end sum” — is useful. Each of these operations involves a number of arbitrary choices, making well-definedness of the resulting manifold a significant question. With regards to the end sum operation, we will discuss familiar situations where all goes smoothly (the notion of semistability plays a role here) and others where significant problems arise. Our analysis leads naturally into the subtle and interesting theory of infinitely generated abelian groups.

**Dubravko Ivanić**

Kirby diagrams of general Cappell-Shaneson 4-spheres

Murray State University

Since the '70s, the topological Cappell-Shaneson 4-spheres have been considered as possible counterexamples to the differentiable Poincaré conjecture in dimension 4. Over the years, increasingly larger subfamilies of these 4-spheres have been shown to be diffeomorphic to the standard 4-sphere, some by simplifying their Kirby diagram (so far available only for a special class), and some by other means.

By representing a Cappell-Shaneson 4-sphere as the result of a side-pairing of the 4-cube, we obtain a Kirby diagram for the general case and simplify it to two 1-handles and two 2-handles. The diagrams for the special class simplify to the standard 4-sphere in a straightforward way, suggesting our approach may be promising for the general case.

**Justin Katz**

Spectral rigidity for certain principal congruence covers of arithmetic surfaces

Purdue University

**Coauthors: D.B. McReynolds**

We will show that isospectrality is equivalent to Gassmann equivalence for commensurable covers of a common surface, provided a certain collection of Selberg  $L$ -functions, associated to a regular cover of the base surface, are multiplicatively independent. We show that when the base surface is associated to a cocompact maximal arithmetic lattice in  $PGL(2, \mathbb{R})$ , and the cover is principal congruence, the relevant  $L$ -functions are indeed multiplicatively independent. We will show how to apply this result to produce infinitely many infinite families of spectrally rigid surfaces.

**Curtis Kent**

Brigham Young University

**Coauthors: Gregory Conner, Wolfgang Herfort, Petar Pavešić**

If a topological space has a universal cover, then every subgroup of the fundamental group corresponds to a covering space. This correspondence allows one to study the fundamental group using covers spaces and vice versa. For spaces, which do not have a universal cover, the situation is more complicated and requires new techniques. One approach is to consider inverse limits of coverings space, which are a special class of unique path lifting fibrations called covering fibrations. We will discuss when a covering fibration is path connected and how this leads to a splitting of the first homology for many spaces.

Covering fibrations

**Noureen Khan**

University of North Texas at Dallas

We discuss a combinatorial method for finding the topological conformation of DNA bound within a protein complex. We use generalized invariants called virtual colorability and search for possible DNA conformations. We apply this method to generalize the classical knot theory cases in order to determine the topological conformation of DNA bound within a stable protein-DNA complex.

Topological conformation of DNA  
bound by virtual coloring

**Matthew Lynam**

East Central University

**Coauthors: Leonard R. Rubin**

In 2012, V. Fedorchuk, using  $m$ -pairs and  $n$ -partitions, introduced the notion of the  $(m, n)$ -dimension of a space. It generalizes covering dimension; Fedorchuk showed that  $(m, n)$ -dimension is preserved in inverse limits of compact Hausdorff spaces. We separately have characterized those approximate inverse systems of compact metric spaces whose limits have a specified  $(m, n)$ -dimension. Our characterization is in terms of internal properties of the system. Here we are going to give a parallel internal characterization of those inverse systems of compact Hausdorff spaces whose limits have a specified  $(m, n)$ -dimension. Fedorchuk's limit theorem will be a corollary to ours.

Inverse systems and  $(m, n)$ -dimension

**Dídac Martínez-Granado**

Indiana University

**Coauthors: Dylan Thurston**

Geodesic currents are measures that realize a closure of the space of curves on a closed surface. Bonahon introduced geodesic currents in 1986, showed that geometric intersection number extends to geodesic currents and realized hyperbolic length of a curve as intersection number with a distinguished geodesic current. Later, other functions were shown to be intersection numbers, such as negatively curved Riemannian lengths (Otal, 1990) or, recently, word length w.r.t. simple generating sets of a surface group (Erlandsson, 2016). Some other functions aren't intersection numbers, such as word length w.r.t. non-simple generating sets or extremal length. In this talk we answer the following natural question: what are sufficient conditions for a function on curves to be an intersection number?

Characterizing intersection numbers

**Michael Mihalik**

Vanderbilt University

A 40+ year old question asks if all finitely presented groups have semistable fundamental group at  $\infty$ . In 1986 the semistability definition was extended to the class of finitely generated groups and

A Finitely Generated Group with  
Non-Semistability Fundamental Group at  $\infty$

successfully used to show certain classes of finitely presented groups have semistable fundamental group at  $\infty$ . Many finitely generated groups have been proposed as candidates to be non-semistable, but up to this point no one has proven that such a group exists. In this talk, we will show that the Lamplighter group does not have semistable fundamental group at  $\infty$ . The technique of proof extends to show all finitely generated groups in a significant class of groups fail to have semistable fundamental group at  $\infty$ . We discuss prospects for finding a finitely presented group with non-semistable fundamental group at  $\infty$ .

**Christian Millichap**

Furman University

Symmetries and Hidden Symmetries of  
Sufficiently Twisted Knot Complements

**Coauthors: Neil Hoffman and Will Worden**

A hidden symmetry of a hyperbolic 3-manifold  $M$  is a symmetry of a finite-sheeted cover of  $M$  that doesn't come from a symmetry of  $M$ . When we restrict to hyperbolic knot complements, hidden symmetries seem to be quite rare though it is difficult to show that an infinite set of knot complements doesn't admit hidden symmetries. In this talk, we will examine how the geometric structures of sufficiently twisted knot complements can be exploited to show that such knot complements have no hidden symmetries and very few symmetries. As a result, these knot complements are usually the only knot complements in their respective commensurability classes.

**Atish J. Mitra**

Montana Tech

The space of persistence diagrams on  $n$  points  
coarsely embeds into Hilbert Space

**Coauthors: Žiga Virk**

We prove that the space of persistence diagrams on  $n$  points (with either the Bottleneck distance or a Wasserstein distance) coarsely embeds into Hilbert space. Such an embedding enables utilisation of Hilbert space techniques on the space of persistence diagrams. We also discuss various non-embeddability results when the number of points is not bounded.

**Molly Moran**

Colorado College

Model  $Z$ -Geometries

**Coauthors: Craig Guilbault**

Bestvina defined a  $Z$ -structure on a group  $G$  to generalize the theory of boundaries of  $CAT(0)$  and hyperbolic groups. An interesting question to ask in this setting is: given two groups that are quasi-isometric where one group is known to admit a  $Z$ -structure, must the other group also admit a  $Z$ -structure? From previous work, we have a partial answer to this question, but there is a roadblock in giving a complete answer. In this talk, we will discuss this roadblock and provide a modified definition of  $Z$ -structures where we hope to give a more complete answer to this question.

**Christoforos Neofytidis**

Ohio State University

Gromov-Thurston norms for circle bundles  
and for fibrations over the circle

A non-negative numerical quantity  $I$  of a closed oriented manifold  $M$  is said to be functorial if whenever there is a map  $f: M \rightarrow N$  then  $I(M) \geq |\deg(f)|I(N)$ . A prominent example is given by Gromov's simplicial volume. We will discuss vanishing and non-vanishing results of functorial numerical invariants for two classes of manifolds: In one direction, we show that there are non-vanishing functorial numerical invariants on each not virtually trivial circle bundle over an aspherical manifold with hyperbolic fundamental group, generalising thus in all dimensions a result of Brooks and Goldmann for circle bundles over hyperbolic surfaces. In another direction, we show

that the simplicial volume of any mapping torus vanishes only in dimensions two and four. Part of this talk is based on joint work with Michelle Bucher.

**Margaret Nichols**  
University at Buffalo

Taut sutured handlebodies as twisted homology products

A basic problem in the study of 3-manifolds is to determine when geometric objects are of ‘minimal complexity’. We are interested in this question in the setting of sutured manifolds, where minimal complexity is called ‘tautness’.

One method for certifying that a sutured manifold is taut is to show that it is homologically simple - a so-called ‘rational homology product’. Most sutured manifolds do not have this form, but do always take the more general form of a ‘twisted homology product’, which incorporates a representation of the fundamental group. The question then becomes, how complicated of a representation is needed to realize a given sutured manifold as such?

We explore the case of sutured handlebodies, and see even among the simplest class of these, twisting is required. We give examples that, when restricted to solvable representations, the twisting representation cannot be ‘too simple’.

**Leonard R. Rubin**  
University of Oklahoma

Čech Systems and Approximate Inverse Systems

**Coauthors: Vlasta Matijević**

The collection  $\text{Cov}(X)$  of open covers of a topological space  $X$  is a directed set with respect to the relation  $\preceq$  of refinement. This means that if  $\mathcal{U}, \mathcal{V} \in \text{Cov}(X)$ , then there exists  $\mathcal{W} \in \text{Cov}(X)$  with  $\mathcal{U} \preceq \mathcal{W}$  and  $\mathcal{V} \preceq \mathcal{W}$ . If we are given that  $\mathcal{U} \preceq \mathcal{V}$ , then a projection  $p_{\mathcal{U}\mathcal{V}} : \mathcal{V} \rightarrow \mathcal{U}$  (not necessarily uniquely defined) is a function such that if  $V \in \mathcal{V}$ , then  $V \subset p_{\mathcal{U}\mathcal{V}}(V)$ . Such a  $p_{\mathcal{U}\mathcal{V}}$  induces a simplicial map  $p_{\mathcal{U}\mathcal{V}} : |N(\mathcal{V})| \rightarrow |N(\mathcal{U})|$ , where  $N(\mathcal{V}), N(\mathcal{U})$  are the nerves of  $\mathcal{V}$  and  $\mathcal{U}$ , and  $|N(\mathcal{V})|, |N(\mathcal{U})|$  are their polyhedra with the weak topology on each. It is known, however, that although projections  $p_{\mathcal{U}\mathcal{V}} : |N(\mathcal{V})| \rightarrow |N(\mathcal{U})|$  and  $p'_{\mathcal{U}\mathcal{V}} : |N(\mathcal{V})| \rightarrow |N(\mathcal{U})|$  can differ, they are always homotopic.

Since  $(\text{Cov}(X), \preceq)$  is a directed set, one might wonder if it is possible to select the projections so that the “Čech system”

$$\mathbf{U} = (|N(\mathcal{U})|, p_{\mathcal{U}\mathcal{V}}, (\text{Cov}(X), \preceq))$$

is an inverse system. This would require that

$$(\dagger) \text{ whenever } \mathcal{U} \preceq \mathcal{V} \preceq \mathcal{W}, \text{ then } p_{\mathcal{U}\mathcal{W}} = p_{\mathcal{U}\mathcal{V}}p_{\mathcal{V}\mathcal{W}}.$$

The first author has proved that for any Hausdorff arc-like space  $X$ , such a  $\mathbf{U}$  cannot even be an approximate inverse system in the sense of Mardešić and Watanabe, i.e., one in which the commutativity relations in the diagrams  $(\dagger)$  are relaxed to certain “gauges of closeness.”

We prove that if  $X$  is a  $T_1$ -paracompactum that contains a nontrivial component, then none of its Čech systems  $\mathbf{U}$  can be an approximate inverse system. This generalizes the result of Matijević because every Hausdorff arc-like space is also a nontrivial continuum, and hence is a  $T_1$ -paracompactum containing a nontrivial component.

**Chandrika Sadanand**  
University of Illinois Urbana Champaign

Heegaard splittings  
and square complexes

A construction of Stallings encodes the information of a Heegaard splitting as a continuous map between 2-complexes. We investigate this construction from a more geometric perspective and find that irreducible Heegaard splittings can be encoded as square complexes with certain properties.

**Nick Salter**

Columbia University

Framed mapping class groups  
and strata of Abelian differentials

**Coauthors: Aaron Calderon**

When a surface  $S$  is non-closed, it can be framed, and the mapping class group acts on the set of framings; the “framed mapping class group” is the stabilizer of a chosen framing. I will discuss work, joint with Aaron Calderon, which gives a generating set for framed mapping class groups “of holomorphic type”. Remarkably, they are generated by finitely many Dehn twists, despite being infinite-index subgroups. One application is to the study of Abelian differentials: we are able to completely describe the monodromy representations of all strata of differentials in genus at least 5.

**Kevin Schreve**

University of Chicago

Towards a Generalized Tits Conjecture for Artin groups

**Coauthors: Kasia Jankiewicz**

The Tits Conjecture (proved by Crisp-Paris) says that the squares of the generators of an Artin group generate the “obvious” right angled Artin subgroup. The Generalized Tits Conjecture says that there should be a larger right angled Artin subgroup generated by high powers of centers of irreducible spherical Artin subgroups. I will explain how the braid group case follows from a result of Koberda, how we can show the conjecture holds for spherical Artin groups not of type  $E_7$  and  $E_8$ , and give some applications.

**Jamie Scott**

University of Florida

Postnikov Essential Manifolds

Let  $X$  be an  $n$ -dimensional CW-complex. Then  $X$  is called essential if the classifying map  $X \rightarrow B\pi_1(X)$  cannot be deformed to the  $(n - 1)$ -skeleton of  $B\pi_1(X)$ . In the context of LS-category,  $X$  is essential if and only if  $\text{cat}(X) = \dim(X)$ ; hence, essential is the defining characteristic for having the maximum possible LS-category according to the dimension bound. But more generally, if  $X$  is  $(r - 1)$ -connected, then we have the bound  $\text{cat}(X) \leq \frac{\dim(X)}{r}$ . However, all essential CW-complexes necessarily have nontrivial fundamental groups, so the above perspective can't be used. We generalize essential to a notion called Postnikov essential in an attempt to get a theorem of the form  $X$  is Postnikov essential of type  $r$  if and only if  $\text{cat}(X) = \frac{\dim(X)}{r}$ . The backwards direction has been proven with only minor adjustments to the original proof, while the forward direction has currently only been proven in special cases.

**Jeremy Siegert**

University of Tennessee Knoxville

Boundaries of coarse proximity spaces  
and their dimension

In this talk we will begin by describing coarse proximities spaces which are structures in coarse geometry meant to capture the notion of “closeness at infinity”. We then describe how to construct a compact Hausdorff boundary on coarse proximity spaces. Many well known examples of boundaries such as the Higson corona and the Gromov boundary arise as boundaries of coarse proximity spaces. Time permitting we will characterize the covering dimension of boundaries using covers within corresponding coarse proximity spaces.



**Bena Tshishiku**

Brown University

Symmetries of exotic negatively curved manifolds

**Coauthors: Mauricio Bustamante**

Let  $N$  be a smooth manifold that is homeomorphic but not diffeomorphic to a hyperbolic manifold  $M$ . How much symmetry does  $N$  have? For example, does  $\text{Isom}(M)$  act on  $N$  by diffeomorphisms? Farrell-Jones showed that in general the answer is “No”; however, they do not rule out that the orientation-preserving subgroup of  $\text{Isom}(M)$  acts on  $N$ . In this talk, we will discuss this problem and its relation to Nielsen realization. The main result is a construction of examples of  $N$  such that  $\text{Isom}(M)$  does act on  $N$ , as well as examples where the largest subgroup of  $\text{Isom}(M)$  that acts on  $N$  has arbitrarily large index.

**Sahana Vasudevan**

Massachusetts Institute of Technology

Distribution of Triangulated Riemann Surfaces  
in Moduli Space

Triangulated Riemann surfaces are compact hyperbolic Riemann surfaces that admit a conformal triangulation by equilateral triangles. They arise naturally in number theory as Riemann surfaces defined over number fields, in probability theory as conjecturally related to Liouville quantum gravity, and in metric geometry as a model to understand arbitrary hyperbolic Riemann surfaces. In many ways, the geometry of triangulated Riemann surfaces mirrors the geometry of arbitrary hyperbolic Riemann surfaces, especially when we consider large genus asymptotics. This suggests that triangulated Riemann surfaces are asymptotically well-distributed in the moduli space of Riemann surfaces. We will describe results in this direction.

**Angela Wu**

Indiana University Bloomington

Weak Tangents of Metric Spaces

A weak tangent of a metric space is the limiting metric space when one zooms in to a metric space. One would wonder how the weak tangents of a metric space is related to the original metric space. In this talk, we explore some of these relationships. We are especially interested in the conformal gauge of weak tangents, i.e. the equivalence classes of weak tangents under quasisymmetry, and their relationships with the conformal gauge of the original class.

## Set-Theoretic Topology

**Dennis Burke**

Miami University

A space  $Z$  paracompact in ZFC;  
CWN screenable Dowker in ZF+AD

Let  $\mathbb{P}$  denote  $\omega^\omega$  and let  $Z = \mathbb{P} \times \omega$ . We construct a topology on  $Z$  which gives a hereditarily normal, collectionwise normal screenable space. This space turns out to be a paracompact D-space in ZFC but is a CWN screenable Dowker space and not a D-space in ZF+AD. This all happens because of a special set  $\Lambda \subseteq \mathbb{P}$  which exists in ZFC but which cannot exist in ZF+AD.

**Christopher Caruvana**

Indiana University Kokomo

The Lie ring of analytic functions  
on a punctured disk

**Coauthors: Robert Kallman**

We show that the topology of uniform convergence on compact subsets is the only Polish topology on the analytic functions defined on a punctured disk which makes them a Polish Lie ring. We also

discuss the complications of extending this result to other domains.

**William Chen-Mertens**

York University

**Coauthors: Paul Szeptycki**

This talk addresses several questions of Feng, Gruenhage, and Shen which arose from Michael's theory of continuous selections from countable spaces. We construct an example of a space which is  $L$ -selective but not  $\mathbb{Q}$ -selective from  $\mathfrak{d} = \omega_1$ , and an  $L$ -selective space which is not selective for a  $P$ -point ultrafilter from the assumption of CH. We also produce ZFC examples of Fréchet spaces where countable subsets are first countable which are not  $L$ -selective.

Selectivity properties of spaces

**Alan Dow**

University of N. Carolina Charlotte

We discuss some recent questions:

1. [Santi Spadaro] Are compact countably tight spaces sequential if they are also pseudoradial?
2. Does ccc forcing preserve Lindelöf for compact first countable spaces?
3. A space is cellular Lindelöf [Bella-Spadaro] if every cellular family traces on a Lindelöf subspace. Does a Lindelöf preserving forcing also preserve cellular Lindelöf?
4. Cellular Lindelöf is not productive, but will the product of a cellular Lindelöf space with any hereditarily Lindelöf space be cellular Lindelöf?

Remarks on tightness, pseudoradiality,  
and cellular Lindelöf

**Jacob Dunham**

University of South Alabama

**Coauthors: Steven Clontz**

For every linearly ordered topological space  $L$ , there is a related topological space  $\check{L} = \{\text{Downward subsets of } L\}$ . Clontz showed that this space is homeomorphic to the Mahavier product  $M\langle 2, \gamma, L \rangle$ . In this talk we will define a partial order topology on any partial order  $P$ , and define two topologies on  $\check{P}$ , both of which turn out to be homeomorphic to  $M\langle 2, \gamma, P \rangle$ .

Downward subsets of partially ordered spaces  
and their relation to Mahavier products

**Todd Eisworth**

Ohio University

We use some elementary topological arguments in the context of pcf theory and obtain some peculiar results in cardinal arithmetic.

Convergence and Pseudopowers

**Jared Holshouser**

University of South Alabama

**Coauthors: Chris Caruvana**

In this talk, we will discuss a method for showing that selection games played on different spaces are equivalent. We will then use this equivalence to generalize previous work connecting a space to its set of real-valued continuous functions.

A Tool for Comparing Selection Games

**Akira Iwasa**

University of South Carolina Beaufort

Consider a continuous map  $f : X \rightarrow Y$ . If  $f$  is an open map, then it remains an open map in any

Preservation of maps by forcing

forcing extension, but if  $f$  is a closed map, then  $f$  may cease to be a closed map in some forcing extension. We discuss under what circumstances a closed map remains a closed map in forcing extensions.

**Peter Nyikos**

University of South Carolina

Applications of the moving off concept

The following concept has appeared in a number of disguises, especially in connection with  $C_p(X)$  and  $C_k(X)$ , the space of continuous real-valued functions on  $X$  with the product and compact-open topologies respectively.

**Definition 1.** Let  $X$  be a nonempty set and let  $\mathcal{K}$  be a collection of nonempty subsets of  $X$ . Then  $\mathcal{K}$  is said to *move off* a collection  $\mathcal{C}$  if for all  $C \in \mathcal{C}$  there exists  $K \in \mathcal{K}$  such that  $C \cap K = \emptyset$ .

For example, two theorems by Gerlits and Nagy [”Some properties of  $C(X)$  I,” Top. Appl. 14 no. 2 (1982)] originally said that  $C_p(X)$  is countably tight [resp. Fréchet-Urysohn] iff every open  $\omega$ -cover has a countable  $\omega$ -subcover [resp. a subcover]  $\mathcal{V}$  such that every point of  $X$  is in all but finitely members of  $\mathcal{V}$ . Recasting this in terms of moving off, we get:

**Theorem.**  $C_p(X)$  is countably tight [resp. Fréchet-Urysohn] iff every collection of closed subsets of  $X$  that moves off the finite sets has a countable subcollection that moves off the finite sets [resp. has an infinite point-finite subcollection].

Other old theorems characterizing when  $C_p(X)$  is Baire, and when  $C_k(X)$  is countably tight or Fréchet-Urysohn, can be similarly ”translated” into the language of moving off. In ”Baireness of  $C_k(X)$  for locally compact  $X$ ” [Top. Appl. 80 (1997), 145-155], Gruenhage and Ma explicitly talked about moving off families, but restricted the concept as follows:

**Definition 2.** A (nonempty) space  $X$  has the *Moving Off Property (MOP)* if every collection of nonempty compact sets that moves off the compact sets has an infinite subcollection with a discrete open expansion.

They showed that a if a (nonempty) space  $X$  has Baire  $C_K(X)$ , then  $X$  has the MOP. It is still an open problem whether the converse holds for Tychonoff spaces, but they showed that it does hold for locally compact spaces.

The new results in the talk concern  $C_0(X)$ , the subspace of  $C_k(X)$  of continuous functions that ”vanish at infinity.” This means that for each  $\delta > 0$ , there is a compact set  $K_\delta$  that contains all the points  $x$  for which  $|f(x)| > \delta$ .

**Theorem 1.**  $C_0(X)$  is countably tight iff every collection of nonempty compact subsets of  $X$  that moves off the compact subsets has a countable subcollection that moves off the compact subsets.

**Theorem 2.**  $C_0(X)$  is Fréchet-Urysohn iff  $X$  has the MOP.

**Strashimir G Popvassilev**

The City College of New York, CUNY,

Medgar Evers College, CUNY, and

Inst. of Mathematics, Bulgarian Academy of Sciences

**Coauthors: Ted Porter**

On some base and monotone covering properties

We discuss some base properties, like base-base paracompactness, and monotone covering properties, like monotone orthocompactness (via open refinements), and state some questions related to them (already published by the authors but remaining open).

**Ted Porter**

Murray State University

Monotone Covering Properties in Scattered Spaces

A question brought up in the work of Chase and Gruenhage's papers on monotonically metacompact spaces is whether monotonically metacompact spaces are the spaces with an Nötherian of subinfinite rank (NSR) basis. Another question is whether Chase and Gruenhage's Property (A) are monotonically metacompact. These questions and others will be considered in scattered spaces. Other monotone covering properties will also be considered.

**Girish Sajjanshetkar**

Mahe Academy of Higher Education

(s, gp) - Open and (s, gp) - Closed functions

In this paper we define (s, gp) - open functions and (s, gp) - closed functions in topology and obtain some of their properties using generalized preopen sets and semiopen sets.

**Alexander Shibakov**

Tennessee Technological University

Ideals in countable groups

**Coauthors: Michael Hrušák**

To study convergent sequences in (mostly) countable groups, we introduce, and prove the consistency of, a new set-theoretic principle we call *the Invariant Ideal Axiom* or *IIA*. We show that the axiom gives uniform negative answers to both Malykhin and Nyikos questions, as well as essentially trivializes the structure of countable sequential groups allowing their complete topological classification.

**Paul Szeptycki**

York University

Bisequential and Absolute Fréchet

**Coauthors: William Chen-Mertens and César Corral-Rojas**

A point  $x \in X$  is an absolute Fréchet point if it remains a Fréchet point in  $\beta X$ . Arhangel'skii introduced the notion of an absolutely Fréchet space in his study of preservation of convergence properties in products. We present some counterexamples to the following questions of Arhangel'skii:

1. Is there a (countable) absolutely Fréchet space which is not bisequential?
2. Is there an  $\alpha_1$ -Fréchet space which is not bisequential?

**Vladimir Tkachuk**

Universidad Autónoma Metropolitana  
de México and Auburn University

Polish cofinality  
and some completeness properties

We prove that, for any cofinally Polish space  $X$ , every locally finite family of non-empty open subsets of  $X$  is countable. It is also established that Lindelöf domain representable spaces are cofinally Polish and domain representability coincides with subcompactness in the class of  $\sigma$ -compact spaces. It turns out that, for a topological group  $G$  whose space has the Lindelöf  $\Sigma$ -property, the space  $G$  is domain representable if and only if it is Čech-complete. Our results solve several published open questions.

**Jerzy Wojciechowski**

West Virginia University

Lineability of the functions that are Sierpiński-Dygmund,

Darboux, but not connectivity

**Coauthors: Gbrel Albkwe and Krzysztof Ciesielski**

Assuming that the continuum  $\mathfrak{c}$  is a regular cardinal, we show that the class  $\mathcal{F}$  of all functions from

$\mathbb{R}$  to  $\mathbb{R}$  that are perfectly everywhere surjective but not connectivity is  $\mathfrak{c}^+$ -lineable, that is, that there exists a linear subspace of  $\mathbb{R}^{\mathbb{R}}$  of dimension  $\mathfrak{c}^+$  that is contained in  $\mathcal{F} \cup \{0\}$ .

Moreover, assuming additionally that  $\mathbb{R}$  is not a union of less than  $\mathfrak{c}$ -many meager sets, we prove the  $\mathfrak{c}^+$ -lineability of the class of functions from  $\mathbb{R}$  to  $\mathbb{R}$  that are Sierpiński-Zygmund, everywhere surjective, but not connectivity.