Southeastern-Atlantic Regional Conference on Differential Equations

Murray State University
Murray, Kentucky, USA

October 19–20, 2007

Contents

Principal Eigenvalue for a Quasilinear Elliptic Problem in an Unbounded Domain

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In this work, we will be concerned with the existence of a positive principal eigenvalue for an nonlinear eigenvalue problem; defined on an unbounded domain. Our approach is the validity on the domain of a Weight Poincare inequlity

Fractional Difference Equations

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Fractional difference equations will be introduced and the method of solving such equations will be discussed by use of R-tansform (discrete Laplace transform). Then we will focus on the existence results for the initial value problems of discrete fractional calculus.

Error Estimation Using Numerical Smoothing Indicator

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We all know Lax-Richtmyer Theorem:

$$\text{Consistency} + \text{Numerical Stability} \Rightarrow \text{Convergence.}$$

In practice, it is hard if not impossible, to verify numerical stability of a scheme while solving an evolution equation, especially if the equation is nonlinear and/or the scheme is complex. A large gap exists between error analysis theory and numerical computation practice. We propose that one can use numerical smoothing to replace numerical stability in error estimate. As a property of a (computed) numerical solution, the numerical smoothing property can be monitored by a "smoothing indicator". This approach has several advantages such as, it works for nonlinear systems and for any time stepping scheme (as long as the smoothing indicator remains bounded). Hence, we can narrow the gap between theory and practice significantly. In this talk, I focus on a finite element solution of parabolic problems.

Uniqueness of Weak Solutions for the Semilinear Wave Equations with Supercritical Boundary/Interior Sources and Damping

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Finite energy solutions of a wave equation with supercritical sources placed in the interior and on the boundary of a bounded domain in $\mathbb{R}^3$ are considered. It is known that local existence of solutions depends on the presence of a superlinear damping. The damping not only extends the life span of solutions, but it is also fundamental in offsetting the lack of locally Lipschitz property (violated in supercritical cases). While existence theory has been in place for some time, the uniqueness of finite energy solutions has been an open problem. The main result presented in this talk is uniqueness and Hadamard well-posedness of finite energy solutions. The class of functions where uniqueness is established contains all the classes for which existence (local in time) is known. As a consequence, the result presented completes the picture of well-posedness of solutions, complementing earlier results on existence due to several authors, including J. Serrin, G. Todorova, E. Vitillaro.

Optimal control of a simple species augmentation model

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The ecological conservation method of ”species augmentation” attempts to reduce species loss by augmenting declining or threatened populations with individuals from captive-bred or stable, wild populations. We use optimal control theory to determine the augmentation strategy for a declining target population

where a growing reserve population is available. An objective functional is formulated to maximize the target population by the final time while minimizing the cost of translocating reserve individuals to the target population. Analysis of numerical simulations examine the effects of varying parameter values and provide conclusions as to the conditions in which augmentation is most effective.

Nonlinear Model Reduction: Methods and Applications

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Model reduction is the process of generating low dimensional systems that approximate behavior of large or infinite dimensional systems. These reduced-order models can then be used in a variety of applications where full-order (or high fidelity numerical simulations) would be prohibitive, such as control, optimization, or gaining better understanding of the underlying dynamics in the system. We begin by discussing the most common method for performing this reduction, proper orthogonal decomposition followed by Galerkin projection. We then present a number of extensions and applications.

Error Estimates for Multiscale Finite Element Methods

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We consider the problem of numerical approximation of the solution of an elliptic PDE whose coefficients have large variations in magnitude over a small spatial scale, as is often the case in porous media flow problems. Multiscale basis functions (rather than standard ones) are used in order to reduce the problem to tractable size. These basis functions will be described and estimates for the error of the numerical solution will be given.

Blow-up of the Solution for Nonlinear Parabolic Problems
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In this talk, we discuss the existence and uniqueness of the classical solution of the following nonlinear parabolic problem,

$$x^q u_t = (u^m)_{xx} + bf(u) \text{ in } (0,1) \times (0,T),$$

$$u(x,0) = u_0(x) \text{ in } [0,1], \; u(0,t) = 0 = u(1,t) \text{ for } t \in (0,T),$$

where $m > 1$, b is a positive number, q is a nonnegative number, $u_0(x)$ is a positive function, and $f(u)$ is a given function. Furthermore, a criterion for u to blow up in a finite time is given.

Optimal Control of an ODE Model for Rabies in Raccoons with a Birth Pulse
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An SEIR model using a system of ordinary differential equations describes a raccoon population infected with the rabies virus. This model includes seasonal births and the dynamics of the vaccine. The goal is to find optimal strategies for distributing vaccine packets to minimize the infected population and the cost of implementing the control. The effect of seasonal birth on this strategy is investigated.

Optimal Control on Hybrid ODE Systems with Application to a Tick Disease Model
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We are considering an optimal control problem for a type of hybrid system involving ordinary differential equations and a discrete time feature. One state variable has dynamics in only one season of the year and has a jump condition to obtain the initial condition for that corresponding season in the next year. The other state variable has continuous dynamics. Given a general objective functional, existence, necessary conditions and uniqueness for an optimal control are established. We apply our approach to a tick-transmitted disease model with age structure in which the tick dynamics changes seasonally while hosts have continuous dynamics. The goal is to maximize disease-free ticks and minimize infected ticks through an optimal control strategy of treatment with acaricide. Numerical examples are given to illustrate the results.

Sensitivity of a Size-Structured Population Model
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We have developed a size-structured population model to assess the growth and development of shrimp populations in both healthy and viral infectious stages. Of major interest is the determination of the sensitivity of solutions of the model with respect to the growth rate, mortality rate, and infection rate of the shrimp. As part of this effort we report on both theoretical and computational findings for the well-known basic Sinko-Streifer population model.

On Compact Support Property of Solutions of Hyperbolic Stochastic Partial Differential Equations

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In this paper we investigate the compact support property of the solutions of hyperbolic SPDE providing that initial condition function is deterministic and has compact support property. First, to approach this problem, we consider semi-SPDE. It turns out that in the semi-SPDE case solution $u(t,x)$ preserve compact support property. When we consider SPDE, we use the stochastic differential-difference equations (SDDE) approach. It turns out that in SPDE case solution $u(t,x)$ does not preserve compact support property. So, if we compare the semi-SPDE and SPDE then it becomes obvious that differentiation in space in SPDE plays crucial role and influence the behavior of the solution.

Liapunov's method and L^p properties of Volterra integral equations.

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Liapunov's direct method has been a popular technique to study various qualitative properties of ordinary and functional differential equations. The use of this method in integral equations, however, is very limited. In this research Liapunov's method is used to study various L^p properties of solution functions of certain nonlinear integral equations of Volterra type.

Third Order Boundary Value Problems on an Unbounded Domain

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We consider the third order nonlinear differential equation

$$(p(t)u'(t))'' = f(t, u(t), u'(t), u''(t)), \quad a.e. \text{ in } (0, \infty),$$

satisfying the boundary condition:

$$u(0) = u'(0) = 0, \quad \lim_{t\to\infty} u(t) = \alpha u(\beta)$$

where $f : [0,\infty) \times \mathbb{R}^3 \to \mathbb{R}$ is Carathéodory with respect to $L_1[0,\infty)$, $p \in C[0,\infty) \cap C^2(0,\infty)$ and $p(t) \geq 0$ for all $t \geq 0$. We obtain the existence of at least one solution using the Leray-Schauder Continuation Principle.

A Second Order Boundary Value Problem with Impulsive Effects on an Unbounded Domain

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We study a second order nonlinear differential equation on an unbounded domain with solutions subject to impulsive conditions and the Sturm-Liouville type boundary conditions. The existence results are obtained via applications of the Krasnosel'skiĭ's fixed point theorem for the sum of a completely continuous operator and a contraction.

Fredholm Integral Equations on Time Scales with Separable Kernels

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Fredholm Integral Equations with separable kernels are considered on Time Scales. Techniques of solving such integral equations as well as existence properties of their solutions are discussed along with some examples.

Hyperbolic Conservation Laws: More Questions than Answers

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The analysis of quasilinear hyperbolic partial differential equations presents a number of challenges. Although equations of this type are important in a number of applications, ranging from high-speed aerodynamics, through magnetohydrodynamics, to multiphase flows important in industrial technology, there is little theory against which even to check the reliability of numerical simulations.

Development of a theory for conservation laws in a single space variable has led to remarkable advances in analysis, including the theory of compensated compactness and the study of novel function spaces. Recently, a number of groups have begun to approach multidimensional systems via self-similar solutions.

In this talk, I will give some history of the development of conservation law theory, including an indication of why the applications are important. I will describe some of the recent results on self-similar solutions, and the interesting results in analysis that they involve. Finally, I will outline some of the paradoxical questions that remain. My research in this area has been joint with Suncica Canic, Katarina Jegdic, Eun Heui Kim, Gary Lieberman, and Allen Tesdall.

Integral equations and initial value problems for nonlinear differential equations of fractional order

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We discuss solvability of integral equations in Lebesgue spaces. The integral equations are associated with initial value problems for a nonlinear differential equation of fractional order. The differential operator is the Caputo fractional derivative and the inhomogeneous term satisfies the Carathéodory conditions and depends on a fractional derivative of lower order.

The Role of the Stability Parameter in a Stabilized Finite Element Method

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Numerical schemes in the context of linear quadratic regulator (LQR) control problems for the convection diffusion equation can be unstable. We develop a stabilization scheme based on the Galerkin Least Squares (GLS) method and compare this scheme to the standard Galerkin finite element method. A careful numerical investigation into the convergence and accuracy of the functional gains computed using stabilization is conducted. We then perform numerical studies of the role that the stabilization parameter plays in this convergence.

Positive Solutions of Singular Second Order Boundary Value Problems on Purely Discrete Time Scales

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We study singular discrete second order boundary value problems with mixed boundary conditions over a finite interval. We prove the existence of a positive solution by means of the lower and upper solutions method and the Brouwer fixed point theorem in conjunction with perturbation methods to approximate regular problems.

Optimal Control applied to a Thoraco-Abdominal CPR Model

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This talk will present optimal control of a circulation model which is discrete in time. The system of seven difference equations models cardiopulmonary resuscitation. The goal is to design external chest and abdomen pressure patterns to improve the blood flow in the heart in CPR procedures. Numerical illustrations will be given.

Bounded Solutions of Nonlinear parabolic Systems

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We consider coupled systems of two nonlinear parabolic equations on a domain which is bounded in space and unbounded in time (namely the entire real line). We establish the existence of bounded solutions existing for all time by using a combination of a priori estimates, comparison techniques, nonlinear approximations, embedding of function spaces, and Gagliardo-Nirenberg type interpolation inequalities which we derive. Additional conditions ensure uniqueness or multiplicity of solutions. Some examples will be given to illustrate the results.

Optimal control applied to native-invasive population dynamics

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This article presents a model for population interactions between an invasive and a native species, where the effect of disturbance in the system (such as flooding) is modeled as a control variable in the growth terms. The motivating example is cottonwood-salt cedar competition, with flooding being detrimental at low and high levels and being advantageous at medium levels, which led us to consider quadratic growth functions of the control. An objective functional is formulated to maximize the native species while minimizing the cost of implementing the control. A new existence result for an optimal control with these quadratic growth functions is given. Numerical results are examined for various parameter values. The results provide suggestions for managing the disturbance regime when invasive species are present.

Boundedness Properties of Solutions of Nonlinear Volterra Integral Equations

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Nonlinear Volterra integral equations of the form

$$x(t) = a(t) - \int_0^t C(t,s)g(s,x(s))\,ds, t \geq 0$$

are studied using the contraction mapping principle as the primary mathematical tool. In particular, the existence of bounded solutions of these equations are found using various boundedness assumptions on $a(t)$ and $a'(t)$.

Spectrum of a function and regularity of solutions to differential equations on Banach spaces.

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In this talk, we first discuss different types of spectrum of a function. Using properties of that spectrum, we next study the regularity of solutions to abstract differential equations of the form $u'(t) = Au(t) + f(t), t \in R$. Especially, the characterization of periodicity and almost periodicity of solutions will be given.

Elliptic Equations with Nonlinear Boundary Conditions

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We will present existence and non-existence results for second order elliptic partial differential equations with nonlinear boundary conditions. The nonlinearity on the boundary interacts in sense with the first eigenvalue of the Steklov spectrum. Our approach is based on a priori estimates and topological degree arguments, and a connection with variational arguments will also be established.

The additional Dynamics of Least Squares Completions for Linear Differential Algebraic Equations

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Many physical problems are most easily initially modelled as a nonlinear implicit system of differential algebraic equations (DAE). However, DAE's contain difficulties that are not present when working with ODE's. There are several approaches proposed for solving higher index DAE's numerically for which the more classical methods may fail. One of these approaches is called explicit integration (EI). The method is based on forming a derivative array by differentiating the DAE a number of times and solving the system using least squares methods. The result is a computed ODE whose solutions contain the solutions of the DAE. This ODE is then integrated using a classical numerical method to get to the soltuions of the DAE. However, the additional dynamics of the least squares completion can affect the numerical soltuions. In this work, we first determine the additional dynamics of least squares completions of linear DAE's and then introduce different modifications of the derivative array for which the additional dynamics do not affect the numerical integration.

A model for the spread of an invasive species

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We will consider the spread of an invasive plant species. At this time we are considering geographic spread of a wind-dispersed invasive shrub, as influenced by human disturbances that provide open-canopied low competition habitat.

We have developed a preliminary hybrid lattice model operating in two stages. The model divides the domain into spatial nodes, each of which has a carrying capacity and intrinsic growth rate for a continuous logistic growth model for plant population. There is then a seed dispersal stage with each lattice node having a probability of plant establishment depending on seed load. The old population is taken as the initial condition in previously occupied nodes and the established seedling population is taken as the initial condition in formally unoccupied node and the logistic stage is run again.

We will show the results of the numerical implementation of the model or this will be an extremely short talk.

The investigators have been partially supported by the Mississippi Computational Biology Consortium (NSF EPSCoR #EPS-0556308).

Stabilization of a nonlinear Schrodinger equation with inhomogeneous boundary value

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In this talk, we prove the decay of energy of solutions of the weakly damped Schrodinger equation with inhomogeneous Dirichlet boundary condition. We prove that if we impose a decaying condition on the boundary condition in a reasonable sense then we get stabilization of the energy. In addition, we observe that decay rate of the boundary function controls the decay rate of energy of the solutions.

Basic and Recent Results in Neutral Differential Equations

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The authors Zhixiang Li, Xiao Wang of the paper

[1] *Existence of positive periodic solutions for neutral functional differential equations*, Electron. J. Diff. Eqns., Vol. 2006(2006), No. 34, pp. 1-8

attempted to prove the existence of positive periodic solutions of the nonlinear neutral differential equation

$$\frac{d}{dt}[x(t) - ax(t-\tau)] = r(t)x(t) - f(t, x(t-\tau))$$

by using cone theory. It was pointed out that the results in the paper are not correct since the two sets Ω_1 and Ω_2 that were constructed by the authors are not open in the Banach space. For the same reason, an addendum has been added to the paper. The main aim of this research is to give a correct proof for the existence of a positive periodic solution by using a different fixed point theorem from the one used in [1].

Neumann Fixed Boundary Regularity for an Elliptic Free Boundary Problem

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We examine the regularity properties of solutions to an elliptic free boundary problem near a Neumann fixed boundary. Consider a nonnegative function u, defined variationally, which is harmonic where it is positive and satisfies a gradient jump condition weakly along the free boundary $\partial\{u > 0\}$. Our main result is that u is Lipschitz continuous. Additionally, we prove various basic properties of such a minimizer near a portion of the fixed boundary on which $\partial u/\partial \nu = 0$ weakly. Our results include up-to-the boundary gradient estimates on harmonic functions with Neumann boundary conditions on convex domains, which have independent interest.

Numerical Solution of Nonlinear Partial Differential Equations of Gas Dynamics and Their Practical Application

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The report covers the typical aspects of numerical solution of nonlinear PDE's of gas dynamics (Navier-Stokes equations) such as selection of initial mathematical model and model of turbulence, selection of the numerical method, verification and testing of developed codes, post-processing and visualization of obtained results. In the capacity of practical application of Navier-Stokes equations, processes of aerodynamics and dynamics of vertical-axis wind turbines were researched. Incompressible Navier-Stokes equations were solved in time-accurate manner using the method of pseudocompressibility and Rogers-Kwak scheme. Three one-equation turbulence models SA, SARC and SALSA were used. Results of numerical simulation for wind turbine rotors with different geometrical characteristics and different number of blades are presented.

A Boundary Layer Basis Method for Fractional Diffusion Equations

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In this talk, we present preliminary analytical and numerical results concerning the finite element approximation of a singularly perturbed fractional advection-dispersion equation (FADE) in one spatial dimension. The FADE is a generalization of the usual advection-dispersion equation in which the underlying stochastic process governing particle jumps is allowed to have infinite variance. We present an error estimate which indicates the under-resolved nature of the singularly perturbed FADE, as well as numerical results from a method in which the usual finite element basis is augmented by a function representing the boundary layer.

Lower Bounds for Blow-up Time in Parabolic Problems

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We consider an initial-boundary value problem for a class of nonlinear parabolic problems in a bounded smooth domain in Euclidean 3-space. We use a first order differential inequality technique on a suitably defined auxiliary function to determine a lower bound on blow-up time if blow-up occurs.

Mathematical Model for Wound Healing

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Wound healing can provide insight into general tissue repair. This talk explores the current state of spatially dependent modeling efforts in the area of wound healing and looks toward the development of a new model. Specifically, we explore the development of a reaction-diffusion model for fibroblast migration in a cylindrical wound to gain a better understanding of the factors influencing successful wound healing.

Positive solutions for a class of p-Laplacian systems with multiple parameters

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Consider the system
$-\Delta_p u = \lambda_1 f(v) + \mu_1 h(u)$ in Ω
$-\Delta_q v = \lambda_2 g(u) + \mu_2 \gamma(v)$ in Ω
$u = 0 = v$ on $\partial\Omega$

where $\Delta_s z = div(|\nabla z|^{s-2}\nabla z), s > 1, \lambda_1, \lambda_2, \mu_1$ and μ_2 are non-negative parameters, and Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$. We prove the existence of a large positive solution for $\lambda_1 + \mu_1$ and $\lambda_2 + \mu_2$ large when

$$\lim_{x\to\infty} \frac{f(M[g(x)]^{1/q-1})}{x^{p-1}} = 0$$

for every $M > 0$, $\lim_{x\to\infty} \frac{h(x)}{x^{p-1}} = 0$ and $\lim_{x\to\infty} \frac{\gamma(x)}{x^{q-1}} = 0$. In particular, we do not assume any sign conditions on $f(0), g(0), h(0)$ or $\gamma(0)$. We also discuss a multiplicity results when $f(0) = g(0) = h(0) = \gamma(0) = 0$.

Approximation of Large Scale Riccati Equations

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Riccati equations are considered for high dimensional problems arising from feedback control of PDE systems. The large number of dimensions prohibit us from solving the Riccati equations directly and we are forced to consider alternative approaches. We consider both Galerkin and Petrov-Galerkin projection methods to reduce the size of the system. The projections are applied to both the high dimensional Riccati equation and the Chandrasekhar equation associated with it. The objective of these projection methods is to obtain low rank approximations to the Riccati solution (and the optimal gain) when the number of control inputs is low. We demonstrate how our methods work using numerical examples arising from the heat equation and an advection diffusion reaction equation (linearized Burgers equation).

Solution Matching for a Second Order Boundary Value Problem on Time Scales

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We will show the existence and uniqueness of solution for a boundary value problem

$$\begin{aligned} y^{\Delta\Delta}(t) &= f(t, y(t), y^{\Delta}(t)) \qquad & t \in [a,b]_{\mathbb{T}} \\ y(a) &= A & y(b) = B \end{aligned} \tag{1}$$

by matching solution of (1) on $[a,c]$ with the solution for (1) on $[c,b]$, where $c \in (a,b)_{\mathbb{T}}$.

Time Periodic Solution of the Korteweg-de Vries Equation on a Bounded Domain and Its Stability

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In this talk we discuss the Korteweg-de Vries (KdV) equation posed on a finite domain with periodic boundary forcings. It is demonstrated that if the amplitude of boundary forcings are small, then the system admits a unique time periodic solution. Moreover it is shown that this time periodic solution is exponentially stable.

From Cell Population Models to Tumour Control Probability Curves: Introduction to Models in Radiation Biology

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In this talk, I will review concepts in radiation biology that allow us to model the effectiveness of radiation therapy in the treatment of cancer. In particular, I will focus on the Tumour Control Probability (TCP), which is the probability that no cancerous cells survive the treatment. Early TCP formulae are based on simple binomial and Poissonian statistics. They are of limited value, since they do not take cell proliferation during the treatment period into account. Recent TCP formulae are based on dynamic models of a cell population, taking cell proliferation as well as the cell cycle into account. I will conclude with a discussion of how and when the TCP formulae are related to each other, and how they can be used to compare the efficacy of different treatment schedules.

Energy balance for dynamic contact with Signorini's condition and slip rate dependent friction.

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Kuttler and Shillor [EJDE vol. 2004, no. 83] show existence results for dynamic frictional contact between a viscoelastic body and a rigid foundation with Signorini conditions for normal contact forces and a nonlocal Coulomb frictional law. However, they do not establish any results regarding energy. We show that their solutions are not only dissipative, but also that the energy lost can be accounted for by viscous and Coulomb friction losses.

Blasting Neuroblastoma Using Optimal Control of Chemotherapy

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The combination of mathematical modeling and optimal control theory provides a strong basis for determining the most beneficial treatment methods for cancer patients. In particular, the effectiveness of new drugs may be analyzed mathematically, generating important data for clinical researchers. In this talk, we will discuss a model developed at St. Jude Children's Research Hospital to investigate the usefulness of the drug Topotecan (TPT) as a treatment for neuroblastoma, a type of cancer that typically targets young children. Optimal control theory is applied to find the treatment schedule that minimizes both tumor volume and the negative effects of chemotherapy.

Existence of Solutions for a Nonlinear Fractional Order Differential Equations

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Let D^α denote the Riemann-Liouville fractional derivative. We are interested in studying the existence of solutions for the nonlinear fractional differential equation,

$L(D)u = f(x, u(x)),\ 0 < x < 1,$
$u(0) = 0,\ u(1) = 0,$
where $L(D) = D^{\alpha_1} - aD^{\alpha_2}$, $1 < \alpha_2 < \alpha_1 < 2$, with $a > 0$.

Nonexistence Results For Classes Of 3x3 Elliptic Systems

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We consider the system

$$\begin{aligned} -\Delta u &= \lambda f(v, w);\ x \in \Omega \\ -\Delta v &= \mu g(u, w);\ x \in \Omega \\ -\Delta w &= \sigma h(u, v);\ x \in \Omega \\ u &= v = w = 0;\ x \in \partial\Omega, \end{aligned}$$

where Ω is a ball in $R^N, N \geq 1$ and $\partial\Omega$ is its boundary, λ, μ, σ are positive parameters bounded away from zero, and f, g, h are smooth functions that are negative at the origin (semipositone system) and satisfy certain linear growth conditions at infinity. We establish nonexistence of positive solutions when two of the parameters λ, μ, σ are large. Our proofs depend on energy analysis and comparison methods.

Backward bifurcation in drug-resistant SIR models

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More and more drug-resistant cases in epidemic diseases are reported by WHO and clinics these years. In addition, vaccination in some childhood diseases are not as effective as expected and the recurrence of certain epidemics seems likely. In this paper, we assume that the recovery period for drug-resistant cases or infection after vaccination is harder and longer, and the vaccination is not fully and permanently protective; we find that the backward bifurcation is very likely and this may explain why some epidemics come back after disappearing for several years.

Controllability and conditioning of POD based models for an optimal control problem.

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We investigate the relationship between proper orthogonal decomposition (POD) bases designed for optimal feedback control problems and the distance to uncontrollability as well as the conditioning of the associated algebraic Riccati equation. As a test problem we consider a reduced order model proposed by Camphouse (R. C. Camphouse, Actuator modes for reduced order modeling and boundary feedback control, Submitted, 2007) for a flow described by the two dimensional Burger's equation. In this work a reduced basis construction method is used that allows for separate consideration of baseline and actuated dynamics in the reduced order modeling process.