Numerical Analysis

MAT 542 – FALL 2010

Homework # 9 Due November 19

1. Verify that

$$x = (2t+1)e^t$$

is the solution to the following boundary value problem:

$$\begin{cases} x'' = x' + x - (2t-1)e^t \\ \\ x(1) = 3e \quad x(2) = 5e^2 \end{cases}$$

2. Verify that

$$x = c_1 e^t + c_2 e^{-t}$$

solves the boundary value problem

$$\begin{cases} x'' = x \\ \\ x(0) = 1 \quad x(1) = 2 \end{cases}$$

if appropriate values of c_1 and c_2 are chosen.

3. The boundary value problem

$$\begin{cases} x'' = 4(x-t), \quad \text{for } 0 \le t \le 1 \\ \\ x(0) = 0 \quad x(1) = 2 \end{cases}$$

has the solution

$$x(t) = \frac{e^2}{e^4-1}(e^{2t} - e^{-2t}) + t.$$

Use the Linear Finite-Difference method to approximate the solution and compare the results to the actual solution. Use $h = \frac{1}{4}$.

4. The boundary value problem

$$\begin{cases} x'' = x' + 2x + \cos t, \quad \text{for } 0 \le t \le \frac{\pi}{2} \\ \\ x(0) = -0.3 \quad x(\frac{\pi}{2}) = -0.1 \end{cases}$$

has the solution

$$x(t) = -\frac{1}{10}(\sin x + 3\cos x).$$

Use the Linear Finite-Difference method to approximate the solution and compare the results to the actual solution. Use $h = \frac{\pi}{8}$.